

ANALYSIS OF THE EFFICIENCY OF STATE PUBLIC EXPENDITURE IN EDUCATION AND HEALTH: THE BRAZILIAN CASE AFTER THE FISCAL RESPONSIBILITY LAW^{1,2}

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The aim of this work was to verify the technical efficiency (i.e., if it is possible use the minimum quantity of inputs required to produce the desired output quantity, given the available technology) of the Brazilian states in relation to public expenditure on health and education after the implementation of the Fiscal Responsibility Law (LRF). For this, a parametric model of stochastic frontier estimation of production and the decomposition of the variation of technical efficiency (VTE) and technological variation by means of the Malmquist index for the Brazilian states were used. Input indicators (expenditure on health and education per capita) were obtained from the National Treasury Secretariat of the Ministry of Finance (Secretaria do Tesouro Nacional/Ministério da Fazenda). Data for output and outcomes in the health area were obtained from Datasus. The data for output and outcomes in the education area were obtained from Ipeadata. The data period is from 2001 to 2012. The results show that, in relation to education expenditures, there is a positive and statistically significant relationship between spending on education per capita and approval rate in secondary education. Regarding the health area, the results show that health expenditures per capita presented a statistically significant and negative relationship with the infant mortality rate in the respective Brazilian states. In relation to the decomposition of the VTE and technological variation through the Malmquist index, all states, both in education and in health, were below the technical efficiency frontier. It should be noted that most of the analyses already carried out of this kind in Brazil focus on the technical efficiency of only the health sector or the education sector separately, while this present work aims to deal with the efficiency of the two sectors.

Keywords: efficiency; public expenditure on health and education; Brazilian states.

ANÁLISE DA EFICIÊNCIA DOS GASTOS PÚBLICOS ESTADUAIS EM EDUCAÇÃO E SAÚDE: O CASO BRASILEIRO PÓS-LEI DE RESPONSABILIDADE FISCAL

O objetivo deste estudo foi verificar a eficiência técnica dos estados brasileiros em relação aos gastos públicos em saúde e educação após a implementação da Lei de Responsabilidade Fiscal (LRF). Define-se a eficiência técnica como o uso de uma quantidade mínima de insumos necessários para produzir a quantidade de produto desejada, dada a tecnologia disponível. Com o intuito de averiguar a eficiência técnica dos gastos em educação e saúde, utilizou-se o índice de Malmquist, um modelo paramétrico de estimação de fronteira estocástica de produção e a decomposição da variação da eficiência técnica e variação tecnológica. Os indicadores de insumo (gastos em saúde e educação *per capita*) foram obtidos na Secretaria do Tesouro Nacional (STN) do Ministério da Fazenda. Dados de produto e resultados na área da saúde foram obtidos no

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Datasus, e os equivalentes para a área educacional foram obtidos no Ipeadata. O período de dados é de 2001 a 2012. Os resultados mostraram que, em relação aos gastos com educação, existe uma relação positiva e estatisticamente significativa entre os gastos com educação *per capita* e taxa de aprovação no ensino médio. Em relação à área de saúde, os resultados mostraram que os gastos com saúde *per capita* apresentaram uma relação estatisticamente significativa e negativa com a taxa de mortalidade infantil nos respectivos estados brasileiros. Em relação à decomposição da variação da eficiência técnica e variação tecnológica por meio do índice de Malmquist, todos os estados, tanto na educação quanto na saúde, estavam abaixo da fronteira de eficiência técnica. Cabe destacar que a maioria das análises já realizadas no Brasil analisa a eficiência técnica apenas do setor da saúde ou da educação em separado, ao passo que este trabalho propõe lidar com a eficiência dos dois setores.

Palavras-chave: eficiência; gastos com saúde e educação; estados brasileiros.

JEL: H21; H51; H52.

1 INTRODUCTION

Since the promulgation of the Brazilian Constitution in 1998, states had received a greater share of public revenue, as a result of the new tributary competencies which they have assumed and of the receipt of a greater share of global tax revenue. A second moment of great impact on the administration of state finances in Brazil was the implementation of the Fiscal Responsibility Law (LRF) of May 4, 2000, which established limits on personnel expenses on 60% of Net Current Revenue (RCL), and to indebtedness (1.2 times the RCL).

LRF was established to guarantee the stability of the Brazilian economy. Until the mid-1990s, the economic environment could be described as of high inflation, high public deficit and excessive debt. To prevent this, LRF has been shown to be a useful mechanism in the search for fiscal balance.⁵ Since then, the idea of responsibility in the administration of public resources has been consolidated in Brazil; governments should not spend beyond what they collect.

In this context, the Brazilian states had to adapt their administrations to a whole new set of rules, involving state financial and budgetary management, as well as conforming to the accountability process performed by governmental oversight agencies and society. Governors began to set fiscal targets and to submit statements of compliance. Moreover, by providing for the integration of the planning and budgeting stages, the LRF opens space for the integration between the financial execution of public spending.

Thus, the paradigm that “spending more is necessarily better” was replaced by the idea that emphasizes what the output of public spending is in relation to its cost. In the case of Brazilian states, which often have limited tax collection capacity and a

5. The work of Giuberti (2005) is recommended for a comprehensive approach to the impact of the LRF.

high dependency on federal transfers, the fulfillment of constitutionally imposed duties depends fundamentally on the appropriate management of their limited resources.

This cost-benefit analysis is particularly essential in investments in education and health, sectors that have considerable importance for the social and economic development of the state, since the main input that modern economies have to grow and develop is the formation of healthy and educated individuals, which in the economic growth literature has been named “human capital” (Schultz, 1960; 1961; Becker, 1962; 1964; Mincer, 1958; 1981). The redefinition of education as investment in human capital, to which the aforementioned authors contributed, and their hypotheses about the relationship between human capital accumulation and aggregate economic growth, formed the basis for arguments that funding for education should be increased, and that the federal government was responsible for providing that increased funding. Therefore, one of the main contributions of this field of research, and an important shift in public policy, is the argument that government has an important role to play in both funding and regulating public education, and that the fundamental purpose of education is to increase future productivity and earnings capacity.⁶

The same argument could be made for health, stemming from Arrow’s (1963) seminal work, which introduced the idea of there being conceptual differences between health and others goods in the economy, the fact that an important economic agent in health, the physician, who makes purchasing decisions, is frequently insulated from prices, and the fact that uncertainty plays an important part. Government intervention, again, comes forth as a crucial aspect in understanding the impact of health in economic development and growth.

In this sense, Grossman (1972) introduced a production function for health, where the author structured a demand model for a specific product, defined as “good health”, in which health problems (mortality and morbidity rates) influence the quantity and productivity of labor. Thus, the Grossman (1972) model allows us to understand that time, the individuals’ income, education, age, among others, influence the production of investments and demand for medical goods and services.

This work, thus, aims to analyze a modern approach to the problem of measuring the quality of state public education and health spending after the implementation of the LRF in Brazil. As a conceptual framework, we use the concept of frontier of efficiency (Coelli et al., 1998; Stead, Wheat and Greene, 2019; Souza et al., 2010) which considers the ability of managers to transform inputs into outputs and results. Our work considered the public policies performed by Brazilian states related to education and health, for which input, output and

6. For an extensive review of the main contributing authors for the theory of human capital, see Holden and Biddle (2016).

outcome indicators were selected. The data was collected from the year 2001 (first year after the implementation of the LRF) until 2012.

In this way, this work sought to answer the question of which Brazilian states have the best quality of expenditure, considering three different flows of the policy: i) states that are more efficient in transforming inputs (here understood as financial resources) into outputs (goods and services made available to the population); ii) more efficient states in transforming inputs (financial resources) into results (more education and health services); and iii) states which are more efficient in transforming outputs (goods and services made available to the population) into results.

In sum, the objective of this work is to verify the technical efficiency of the Brazilian states in relation to public spending on health and education after the implementation of the LRF. It should be noted that most of the analyses already carried out in Brazil focus on the technical efficiency of only the health sector or the education sector separately.

In formal terms, we estimate the total factor productivity (TFP) and its decomposition in terms of the variation of technical efficiency (VTE) and technological variation using the Malmquist index for Brazilian states, in its aggregate form, using the parametric model of stochastic estimation of the frontier of production. Two basic assumptions were made about the characterization of the production function: the Cobb-Douglas production function and the translog production function. Lastly, we compare, for both education and health, the efficiency scores between Brazilian states.

This work is divided into five sections beyond this introduction. Section 2 presents a literature review. Section 3 presents the methodology used. Section 4 presents the results obtained with the estimates. Lastly, the concluding remarks.

2 LITERATURE REVIEW

2.1 Productivity and efficiency

The production models used most often consider a technology that produces a single product, which is commonly described by a production function, $f(x)$. However, the properties of the production set validated by economic theory apply directly to the production function.

In view of the above, it is then possible to define the production function or production frontier, $f(x)$, as that which describes the technical relationship between the inputs and the outputs of a production process, defining the maximum output attainable from of a given input vector, that is:

$$f(x) = y \in R: y \text{ is the maximum output associated with } -x \text{ in } y\}. \quad (1)$$

Considering the diverse types of production functions, the Cobb-Douglas and the translogarithmic functions can be listed as two of the main ones. The first is a type of function widely used in microeconomic analysis, and can be described as follows:

$$y = f(x_1, x_2, \dots, x_n) = a_0 x_1^{b_1} x_2^{b_2} \dots x_n^{b_n} \quad (2)$$

The Cobb-Douglas function is easy to estimate and manipulate mathematically, but it is restrictive in relation to the properties imposed on the production structure (the value of the returns to scale and the elasticity of substitution is equal to the unit). Formulation of the Cobb-Douglas function is often not satisfactory to represent certain production processes. In such cases, it is common to adopt a more flexible formulation, adjusting to a function of the translogarithmic type (or translog), namely:

$$\ln y = a_0 + \sum_{i=1}^n a_i \ln x_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n b_{ij} \ln x_i \ln x_j \quad (3)$$

The translog function does not impose restrictions on the production structure, but it is more difficult to manipulate mathematically, as it reduces the degrees of freedom and presents problems of multicollinearity.

Works on productivity and efficiency in economics began with the work of Farrel (1957), who focused on measuring efficiency according to the use of inputs. One can, thus, examine the sources of productivity growth over time and the differences in productivity between countries and regions.

Productivity growth can be defined as the net change in output due to changes in efficiency and technological changes, where the former is the variation of the output observed in relation to its frontier, and the latter represents the displacement of the frontier of production (Tupy and Yamaguchi, 1998). Rivera and Constantin (2007), stated that not all producers are technically efficient, that is, not all producers are able to use the minimum quantity of inputs required to produce the desired output quantity, given the available technology. In general, research scholars use index numbers to analyze productivity growth without distinguishing the factors that caused the changes, that is, if this increase occurred only due to the increase in the use of inputs, or if there were variations in technical efficiency or technological variations (Carvalho, 2003).

Both efficiency and productivity are indicators of success, performance measures, through which productive units are evaluated. This performance, in turn, is a function of two factors: the state of the technology and the degree of efficiency of its use. The technology defines the frontier relationship between inputs and outputs, while the efficiency incorporates waste and misallocation of resources related to this frontier (Carvalho, 2003).

Farrel (1957) states that the study of efficiency and productivity has become important because this is the main step in a process that can lead to substantial savings in resources, which is of great importance for companies in competitive environments. Thus, the relationship between the quantity produced and the quantity used of inputs can be used as a measure of the firm's performance. The highest values of this ratio are associated with the best performances (highest productivity).

Farrel (1957) argues that, in the traditional approach, the measure of productivity assumes that the production obtained results from the best practice or is the frontier production (maximum production possible, given the quantities of inputs used). This assumption assumes that the production observed throughout the period is technically efficient in the sense of Farrel (1957). On the other hand, in contrast to the traditional approach, the frontier approach to measuring productivity explicitly incorporates inefficiency and computes changes in efficiency (Grosskopf, 1993).

When there is an increase in productivity of the production unit between two periods of time, this increase can be attributed to technological variations, efficiency gains, exploitation of economies of scale or the combination of all these factors. Therefore, variations in technical efficiency are understood as changes in the distance of the observed product in relation to its border. Therefore, technological variations involve advances in technology, which can be represented by an upward shift in the production frontier.

2.2 Measuring efficiency

Among the most common methods for measuring the efficiency of production units, two of them stand out in most studies on productivity and efficiency. The first is the statistical (or econometric) approach, and the second is the mathematical (deterministic) approach, being the two approaches quite different.

Segunpta (1999) states that the econometric approach, for using a certain functional form for the function (for example, Cobb-Douglas), is parametric, that is, the estimation of the efficiency of the production technology is done through parameters, tested from standards. However, there are some disadvantages to using this approach. The error of bad specification can occur, often caused by the functional form of the production function. Furthermore, the measurement of efficiency in which there are multiple inputs and multiple products is not easily applied in a parametric model. In this sense, the most appropriate approach is the Data Envelopment Analysis (DEA). In the DEA approach, linear programming methods are used to calculate the efficiency boundary. Consequently, this approach avoids misspecification errors.

The disadvantages of the DEA model, according to Geva-May (2001), is that, first, it does not provide estimates or tests of significance of the parameters; second, the envelope boundary can be defined only for small samples. Other factors can influence the results obtained through the DEA approach, such as, for example, the heterogeneity of the production units and the fact that the DEA considers that all deviations to the frontier are treated as inefficiency.

2.3 International evidence

A traditional approach to assessing the differences in efficiency of a production units is to use input and output indicators, with their respective prices, and analyze their productivity as the ratio of their weighted outputs and their weighted inputs. The market prices of inputs and outputs are used as weighting factors when available. One of the basic problems of evaluating the efficiency of the public sector is that market prices for its outputs are not available, as in the case of basic education offered by state schools. Another strategy is to first estimate the frontier of production and derive the efficiency scores based on the relative distance of the inefficient observations to the frontier. Then the efficiency scores are explained by regression models.

In addition to parametric techniques to establish the aggregate production function where multiple outputs are weighted, non-parametric techniques have been used to establish the frontier of production. Its virtue lies in the fact that the production frontier can be derived in multiple outputs and multiple inputs without the need to attribute weight to the variables. The efficiency scores can thus be explained by the characteristics of the states in the regression models.

An application of the above strategy was used in De Borger et al. (1994), when studying the technical efficiency of 589 Belgian municipalities with cross-section data for the year 1985. Firstly, the authors used a non-parametric methodology called Free Disposal Hull (FDH), which is an application of linear programming. From that, the frontier of production and the efficiency scores for the municipalities were derived.

Borger and Kerstens (1996) compare results of parametric and non-parametric applications in the efficiency of municipal services, using the same data from municipalities in Belgium. The authors compare two non-parametric methods (FDH and DEA) and three parametric methods (one deterministic and two stochastic). Although the results of the ranking of efficiency scores were different, in all models there is a positive relationship between the average educational level and the size of the municipalities.

Afonso and Scaglioni (2005) evaluated the efficiency of the different regions in Italy concerning the provision of strategic public services such as general administration, water and sewage, energy, transport and solid waste disposal for 2001. It should be noted that the Italian context analyzed in that work resembles the Brazilian scenario post-LRF, as it addresses an instant in time of transition in Italy, moving from a highly interventionist state presence to a modern regulatory environment, focused on transparency. In order to carry out such an analysis, the authors constructed an efficiency index for the provision of public services in the different Italian regions and through the methodology of DEA, and conclude that there is a considerable discrepancy in the level of efficiency of such provision between the localities, and that, on average, some regions could increase their output by 28% using the same resources as the most efficient regions.

In a more comprehensive work, Herrera and Pang (2005) analyze the efficiency of public spending in developing countries using the efficiency frontier approach. The sample consists of 140 countries, with data from 1996 to 2002, and the methodology uses both FDH and DEA to measure efficiency. The results indicate that countries with higher expenditures have lower efficiency levels, as well as countries which have the payroll as being the largest part of the public budget. The authors also find that countries with higher levels of inequality, greater reliance on foreign aid and localities suffering from acquired immunodeficiency syndrome (AIDS) epidemics also tend to be less efficient in public spending.

Afonso and Aubyn (2005) address the issue of efficiency in education and health spending in the context of the Organisation for Economic Co-operation and Development (OECD) countries using the non-parametric methodologies of FDH and DEA. Analyzing data for the year 2000, the authors find results that suggest that the efficiency of public sector spending in these two important sectors varies for education in terms of input efficiency between 0.859 and 0.886, depending on the methodology used, and for health, the values vary between 0.832 and 0.946.

Thus, in less efficient countries, there is room for better results using the same level of resources. South Korea, Japan and Sweden were the most efficient countries, regardless of the method used. South Korea is particularly efficient in terms of education, and Japan is the most efficient in terms of both education and health, and it should be noted that when comparing Sweden and Japan, some interesting insights can be pointed out, as to how it is possible be efficient in different ways. For example, Japanese students spend more time in the classroom than the Nordics, and in terms of health, while Japan does not have such a large quantity of doctors, the country does feature an abundance of hospital beds.

TABLE 1
International literature synthesis

Author	Indicators	Data and period	Method	Results
De Borger et al. (1994)	Education	589 Belgian municipalities, 1985	FDH and Tobit models	Most efficient units are positively related to the size of the municipality and the average educational level among the population. The average income level and the proportion of aid on municipal revenue are negatively related to efficiency.
De Borger and Kerstens (1996)	Education	589 Belgian municipalities, 1985	The authors compare two non-parametric methods and three parametric methods	Positive relationship between the average educational level and the size of the municipalities.
Afonso and Scaglioni (2005)	General administration, water and sewage, energy, transport and solid waste disposal	Different regions of Italy, 2001	DEA	Discrepancy in the level of efficiency of provision of public goods and services between the localities, on average, some regions could increase their output by 28% using the same resources as the most efficient regions.
Herrera and Pang (2005)	Health and education	140 countries, 1996-2002	FDH and DEA	Countries with higher expenditures have lower efficiency levels, as well as countries which have the payroll as being the largest part of the public budget. Countries with higher levels of inequality, greater reliance on foreign aid and localities suffering from AIDS epidemics also tend to be less efficient in public spending.
Afonso and Aubyn (2005)	Education and health	OECD countries, 2000	FDH and DEA	Efficiency of public sector spending in education and health varies for education in terms of input efficiency between 0.859 and 0.886, depending on the methodology used, and for health, the values vary between 0.832 and 0.946.

Authors' elaboration.

In this work, the output of municipal public services is measured considering three inputs (number of skilled workers, number of unskilled workers and the physical area of public buildings) and five specific outputs. The latter measures refer to the extension of public roads, number of municipal aid beneficiaries, number of students enrolled in basic education, extension of public recreation equipment and a proxy for services provided to non-residents. In a second stage, when the municipal efficiency scores are explained by Tobit models, it was found that the most efficient units are positively related to the size of the municipality and the average educational level among the population. The average income level and the proportion of aid on municipal revenue are negatively related to efficiency.

2.4 Evidence for Brazil

For applications in Brazil, the majority of works aims to measure the efficiency of hospitals or schools, and more recently, the provision of public services by municipalities. In this line of research considering municipal expenditures, Marinho (2001), for example, analyzes outpatient and hospital services in the municipalities of the state of Rio de Janeiro. The author uses non-stochastic efficiency frontiers and, in combination with regression models, and, firstly, observes that in relation to the efficiency scores, there is a great dispersion of results among the municipalities, with the general average being 83%, that is, a medium level of efficiency. Regarding the resources, it can be noted that the greatest imbalances are in the number of hospitals per capita, which is 25.7% above the optimal value, and in ambulatory capacity per capita, which was 24.3% above the optimum, that is, on average the number of units is approximately 25% greater than the optimum. The mismatches observed in the system, according to the author, are mostly related to the excessive number of units, rather than the actual capacity of the outpatient and hospital services network in the state.

Bezerra and Diwan (2001) perform several analyses using DEA models that consider as outputs the indicators that make up the Human Development Index (HDI), with the intention of proposing an alternative and complementary method to this measure of human development. The focus is on indicators of social areas such as education, health and sanitation for the largest Brazilian municipalities in 1996. The authors find interesting results, such as the fact that some low HDI locations have a high level of efficiency, while some municipalities have high expenditures on education and health, but have unsatisfactory levels of efficiency since they produce similar results to municipalities with more modest expenditures.

Faria (2005) analyzes the relationship between social expenditures in municipalities – that is, public spending on education and culture, health and sanitation – and indicators of the living conditions of the population living in them, in the 1990s, using data from the 2000 Census of the municipalities of

Rio de Janeiro, through the application of the DEA technique. The author finds a group of four municipalities that present maximum efficiency in the health and sanitation model, as well as in the model of education and culture. However, a more significant number of municipalities appear to have low levels of efficiency in the provision of these various public services.

The work developed by Brunet et al. (2006) analyzes the efficiency of state public spending in Brazil by budget functions using the FDH Performance Frontier model, as well as using the Hill Function Adjustment Model to study the functions of education and culture, health and public safety for the 26 Brazilian states and the Federal District. When observing the results obtained in each of the budget functions analyzed, the conclusion is that, in general, states with lower levels of input (i.e., lower expenditure per capita) perform better in terms of efficiency and effectiveness of public expenditure. However, lower levels of global expenditure imply a lower supply of outputs or services and fewer results, leading to the lower general welfare of the population (Brunet et al., 2006). More specifically, through the adjustment of the Hill Function, the authors note that the states considered efficient in education and culture are Roraima, Rondônia, Pernambuco and Mato Grosso. Considering efficiency in the provision of health services, the states which stand out are Maranhão, Piauí and Rio de Janeiro, and Paraíba and Federal District are the states that do not appear in the frontier of efficiency.

Faria, Jannuzzi and Silva (2008) developed an analysis of public policies when verifying if the budgetary resources for the 62 municipalities in Rio de Janeiro are well applied, that is, if education and health expenses reflect improvements in the respective indicators. For that purpose, the DEA technique was used to perform an evaluation of the efficiency of social spending, and for that, the available resources were used as inputs and as outputs the results, or rather, the impact of the services provided. Data consisted of social indicators of municipal expenses in the following areas: education and culture; health and sanitation. The authors review several DEA applications to public policies and then present the indicators and models considered for social expenditure efficiency analysis. The Banker, Chernes and Cooper (BCC) output model of DEA was chosen, so as to maximize outputs without decreasing inputs. Indicators of expenditure per capita on education and culture and on health and sanitation were used as inputs of the model in question. The output indicators are the literacy rate of 10-14 year-old children; the ratio of permanent private homes with adequate sanitary disposal; the ratio of permanent private homes with proper sanitation; the inverse of mortality rate by sanitation causes; the ratio of 2-5 year-old children registered at day child-care centers or children schools. The analysis shows that the municipalities of São Gonçalo, Japeri, Queimados, Cantagalo, São João de Meriti, and Resende as being efficient, due to the results these places achieve in terms of either of resource allocation or of medium revenue.

Machado Júnior, Irffi and Benegas (2011) sought to evaluate the technical efficiency of municipal per capita expenditures on education, health and social assistance for the municipalities of Ceará, considering data for 2005. The authors adopt the methodology of DEA. Health data uses the municipal health expenditure, mortality rate, water supply rate, and basic sanitation coverage. For education, the data refers to the number of kindergartens, the rate of schooling and the rate of child literacy. Different models are proposed, focusing on the efficiency of public spending in the three areas, as well as specific models for each sector. In general, when analyzing public expenditures jointly by a model that contemplates the three areas, the results of the model seem to indicate that the municipalities of the state of Ceará have performed satisfactorily. When, however, the models which consider the type of specific expenditure were estimated, the results show low efficiency of public spending per capita.

Lima, Moreira and Souza (2014) evaluated the level of efficiency of public spending in the Brazilian states for the reduction of poverty in the period from 2004 to 2009. For this, a fixed-effect model and a stochastic frontier model were used. Of particular interest, in addition to studying which factors contribute to poverty reduction in the country, the authors seek to evaluate whether, specifically, expenditures on education, health, sanitation, care, security and transportation are used efficiently in the fight against poverty. The empirical results are consistent with most of the literature and point to the fact that the performance level of economic activity is fundamental for decreasing of poverty, that is, higher employment and higher income per capita contribute significantly to the reduction of the rate of poverty in the country. The efficiency analysis shows that spending on transfers to poor families is inefficient in fighting poverty, but retirement spending is efficient. However, expenditures per capita on education and culture, health and sanitation, public security and transportation do not affect efficiency or inefficiency in the process of combating poverty, since they presented statistically zero coefficients. These results, according to the authors, may indicate that there is a poor allocation of resources.

Mazon, Mascarenhas and Dallabrida (2015) sought to evaluate the technical efficiency in the use of the Unified Health System (SUS) resources in seven municipalities in the state of Santa Catarina. The DEA methodology was applied and the Health Technical Efficiency Index (Iets) proposed by Mendes was calculated. The results show that all municipalities presented low Iets and only one municipality showed adequate technical efficiency.

Barbosa and Souza (2015) analyzed, using DEA, the technical efficiency and scale of the Performance Index of the SUS (IDSUS) of the municipalities of the Brazilian Northeast. The results indicate that the Northeastern municipalities with the highest indices referring to the specificities presented, on average, the best scores of technical efficiency and scale of the IDSUS in the municipalities of the Northeast of Brazil.

TABLE 2
Literature review considering Brazil: synthesis

Author	Indicators	Data and period	Method	Results
Marinho (2001)	Health	Rio de Janeiro, 1998	Non-stochastic efficiency frontiers and regression models	Dispersion of results among the municipalities, with the general average being 83.07%, that is, a medium level of efficiency. Imbalances in the number of hospitals per capita, which is 25.7% above the optimal value, and in ambulatory capacity per capita, which was 24.3% above the optimum.
Bezerra and Diwan (2001)	Education, health and sanitation	98 Brazilian municipalities, 1996	DEA	Low HDI locations have a high level of efficiency, while some municipalities have high expenditures on education and health, but have unsatisfactory levels of efficiency since they produce similar results to municipalities with more modest expenditures.
Faria (2005)	Education and culture, health and sanitation	Municipalities in the state of Rio de Janeiro, 2000	DEA	A group of four municipalities present maximum efficiency in the health and sanitation model, as well as in the model of education and culture. A significant number of municipalities appear to have low levels of efficiency in the provision of these various public services.
Brunet et al. (2006)	Education and culture, health and public safety	Brazilian states, 2002 to 2004	FDH and Hill Function Adjustment Model	States with lower levels of input (i.e., lower expenditure per capita) perform better in terms of efficiency and effectiveness of public expenditure. Lower levels of global expenditure imply a lower supply of outputs or services and fewer results, leading to the lower general welfare of the population.
Faria, Jannuzzi and Silva (2008)	Education and health	62 municipalities in Rio de Janeiro, 1999-2000	DEA	Municipalities of São Gonçalo, Japeri, Queimados, Cantagalo, São João de Meriti, and Resende as being efficient, due to the results these places achieve in terms of either of resource allocation or of medium revenue.
Machado Júnior, Irfri and Benegas (2011)	Education, health and social assistance	Municipalities in the state of Ceará, 2005	DEA	The results of the model seem to indicate that the municipalities of the state of Ceará have performed satisfactorily. When, however, the models which consider the type of specific expenditure were estimated, the results show low efficiency of public spending per capita.
Lima, Moreira and Souza (2014)	Education, health, sanitation, care, security and transportation	Brazilian states, 2004 to 2009	Stochastic frontier model and fixed effects model	Spending on transfers to poor families is inefficient in fighting poverty, but retirement spending is efficient. However, expenditures per capita on education and culture, health and sanitation, public security and transportation do not affect efficiency or inefficiency in the process of combating poverty, since they presented statistically zero coefficients.
Mazon, Mascarenhas and Dallabrida (2015)	Health	Seven municipalities in Santa Catarina, 2010	DEA and lets	The results show that all municipalities presented low lets and only one municipality showed adequate technical efficiency.
Barbosa and Souza (2015)	Health	1790 Municipalities in the Northeast region of Brazil, 2012	DEA	The results indicate that the Northeastern municipalities with the highest indices referring to the specificities presented, on average, the best scores of technical efficiency and scale of IDSUS in the municipalities of the Northeast of Brazil.

Authors' elaboration.

Thus, considering all the evidence previously mentioned, one can observe that there are several studies analyzing the efficiency of public spending of health and education, both in the international literature, as well as in works considering Brazil. However, particularly for the case of Brazil, it is rare to find works which consider both health and education in their analyses simultaneously, which is the main goal and contribution of this current work. For that purpose, in the following section the methodology used is presented and explained thoroughly.

3 METHODOLOGY

3.1 Data

This work consider the spending from public policies performed by Brazilian states related to education and health, for which input, output and outcome indicators were selected. The data are collected from the year 2001 (first year after the implementation of the LRF) until 2012. Input indicators (expenditure on health and education per capita) were obtained from the STN. Data for output and outcomes in the health area were obtained from Datasus. The data for output and outcomes in the education area were obtained from Ipeadata.⁷

These health indicators are used by the Ministry of Health to visualize the spatial structure of health in the federal units of Brazil, regarding the infrastructure of resources (number of beds, number of doctors, number of nurses), the coverage (coverage rate of the Family Health Program⁸ and health expenditure per capita), and results (infant mortality rate).⁹

Specifically, the selected indicators in the health area are: input indicators (health expenditure per capita – *Gastsaud*); output indicators (number of health beds per 1,000 inhabitants – *Leitos*, number of doctors per 1,000 inhabitants – *Med*; number of nurses per 1,000 inhabitants – *Enf*; PSF coverage rate – *Txcob*); outcome indicators (infant mortality rate per thousand live births – *Txmort*).

Table 3 presents the descriptive statistics of the series used in the health area. One can observe that, on average, the states spent R\$ 171.11 per capita on health during the period analyzed. On average, there is also 1 hospital bed per 1,000 inhabitants, 1.75 doctor and 0.87 nurses for every 1,000 inhabitants. The infant mortality rate in the analyzed period was 20.88 for every 1,000 live births. The coverage rate of the PSF was, on average, 53.18% of the population of the states.

7. The choice of input and output variables for spending on education and health was based on the study by Machado Júnior, Irfi and Benegas (2011).

8. Programa Saúde da Família (PSF).

9. For more information, see: <<https://bit.ly/3jzFx7u>>. Accessed on: Dec. 7, 2022.

TABLE 3
Descriptive statistics of the series in the health area (2001-2012)

Variables	Observations	Mean	Standard deviation	Minimum	Maximum
<i>Gastsaud</i>	324	171.1100	124.9308	6.01	627.37
<i>Leitos</i>	324	1.0090	0.4358	0.19	2.42
<i>Med</i>	324	1.7532	7.0727	0.10	128
<i>Enf</i>	324	0.8769	0.5603	0.08	4.04
<i>Txmort</i>	324	20.8893	6.2827	9.39	97.36
<i>Txcob</i>	324	53.1805	21.6733	3.21	36.72

Sources: Datasus (available at: <<https://bit.ly/3YQqeaJ>>; accessed on: Dec. 7, 2022); and STN (available at: <<https://bit.ly/3Vqoz8X>>; accessed on: Dec. 7, 2022).

Authors' elaboration.

The education indicators are variables used to analyze and measure the educational level of the states,¹⁰ as well as to verify the existing structure in the state education networks and the results obtained. Thus, these are: input indicators (expenditure with education per capita – *Gastedu*); output indicators (number of teachers per student enrolled in the state education network – *Prof*); number of schools – *Escola*); indicators of outcome (High School approval rate – *Txaprov*).

Table 4 presents the descriptive statistics of the education series. It can be observed that in the analyzed period, the states spent an average of R\$ 332.33 with education. The average number of teachers in the secondary education network was 15,736 professionals. The average number of schools was 631.54. The approval rate in secondary education was 70.89%.

TABLE 4
Descriptive statistics of the series in the area of education (2001-2012)

Variables	Observations	Mean	Standard deviation	Minimum	Maximum
<i>Gastedu</i>	324	332.3356	213.0388	65.9068	1,324.76
<i>Prof</i>	324	15,736.77	20,857.77	263	116,549
<i>Escola</i>	324	631.5401	735.2853	47	3,986
<i>Txaprov</i>	324	70.8987	6.9070	52.12	87.20

Sources: Ipeadata (available at: <<https://bit.ly/3W1YfYJ>>; accessed on: Dec. 7, 2022); and STN (available at: <<https://bit.ly/3Vqoz8X>>; accessed on: Dec. 7, 2022).

Authors' elaboration.

3.2 Stochastic frontier model

As featured and discussed in Stead, Wheat and Greene (2019), the stochastic frontier model is typically used to estimate best practice ‘frontier’ functions that explain

10. Available at: <<http://bit.ly/3WFHMTV>>. Accessed on: Dec. 7, 2022.

production or cost and predict firm efficiency relative to these. A key feature of these models, as argued by the authors, is the focus on unobserved disturbance in the econometric model. This entails a deconvolution of the disturbance into a firm inefficiency component, where the quantification of which is the goal of the analysis, and a statistical noise term (Stead, Wheat and Greene, 2019).

The stochastic frontier approach allows for shifts from the frontier to represent both inefficiency and an unavoidable statistical noise, with the aim of being a closer approximation to reality, since observations usually involve random errors.

Coelli et al. (1998) define the production function of a production unit i in period t as:

$$y_{it} = \exp(x_{it}\beta + v_{it} - u_{it}). \quad (4)$$

This production function can be rearranged in the following way:

$$y_{it} = \exp(x_{it}\beta + v_{it}) \exp(-u_{it}), \quad (5)$$

or

$$\ln y_{it} = x_{it}\beta + v_{it} - u_{it}, \quad (6)$$

in which:

- y_{it} is the vector of produced quantities (*outputs*);
- x_{it} is the vector of inputs used in production (*inputs*); and
- β is the vector of coefficients to be estimated (*parameters*).

The terms v_{it} and u_{it} are vectors representing different components of the error term. The first one refers to the random part of the error, with normal distribution, independent and identically distributed, truncated at zero and with constant variance σ^2 , ($v \sim \text{iid } N(0, \sigma^2)$), while the second term represents technical inefficiency, that is, the part that constitutes a downward deviation from the frontier of production, which can be inferred by the negative sign and by the restriction $\mu \geq 0$. They are non-negative random variables with normal distribution truncated at zero, independently distributed (not identically) with mean μ_{it} and constant variance σ_u^2 , i.e., ($u \sim \text{NT}(\mu, \sigma_u^2)$). The error components are independent of each other and x_{it} is supposed to be exogenous, so the model can be estimated by the maximum likelihood technique (Araújo, Feitosa and Silva, 2014).

The maximum likelihood function was reparameterized in terms of the parameter $\gamma = \sigma_u / (\sigma_u + \sigma_v)$ so that the effects of the technical inefficiency of the model can be verified by means of statistical tests. If γ is considered statistically

equal to zero, there is no influence of the technical inefficiency in the model, thus being possible to apply ordinary least squares for the estimation of the parameters. The closer γ is to one, the greater the importance of technical inefficiency in the model.

The main advantage of considering a stochastic frontier analysis is that, unlike other methods, it introduces an error component to represent noise, measurement errors. Moreover, it allows the decomposition of the deviation of an observation into two components: random noise and the effects of technical inefficiency of production (Carvalho, 2003).

Coelli et al. (1998) justify the choice of parametric models by means of some properties, which are:

- the possibility of performing hypothesis tests on the parameters of the explanatory variables;
- the possibility of including control variables to explain technical inefficiency in only one stage; and
- allows the presence of random noise in the environment in which the decision-making unit operates.

For Souza et al. (2010), although the stochastic frontier of production model presents the advantages described in the properties summarized by Coelli et al. (1998), this model requires the imposition of an a priori functional form and the performance of a test of hypotheses concerning the distribution of the term of inefficiency. Also, according to Pal (2004), the question of relation between the noise variable and the inefficiency error can be raised. The random shock leading to higher output could influence management to become more efficient. Nevertheless, it is possible to test if there is any correlation between the two error components.

Nonparametric models, such as the DEA model, do not require such restraints. When assuming constant returns of scale, as is the case of the present work for the reasons explained in the following section that discusses the test for the functional form, the two methodologies, stochastic frontier of production and DEA, provide the same result,¹¹ so this work will focus on using the first methodology only. The main objective of the model is the estimation of a production function in terms of its factors of production. In this sense, the maximum production can be obtained by employing a certain efficient combination of factors. However, there is no guarantee that an efficient combination of factors will be used to maximize production. Considering the possibility of technical inefficiencies, the results can be featured below the maximum frontier of production. Hence the origin of the stochastic production function model.

11. As discussed, for example, in Afonso and Aubyn (2005).

3.2.1 Hypotheses tests

Functional form test

Some hypothesis tests concerning the estimation of the stochastic frontier are relevant, since the estimation is based on the statistical (econometric) approach. As described in previous sections, parametric estimation of the frontier requires that the production function assume a functional form, which can be determined by means of the generalized likelihood (LR) ratio test. Initially, the production function is estimated in the Cobb-Douglas form and then in the translog form, so that it is possible to compare the two functions through the functionality test, in order to identify the more appropriate functional form to be used in the model. This procedure is well defined in recent works by Araújo, Feitosa and Silva (2014).

The functionality test aims to verify the null hypothesis that the Cobb-Douglas form is the most appropriate functional form to represent the data, given the specifications of the translog function. After the estimation of the two models previously mentioned, the respective log-likelihood (LL) values are verified, and from the value of the LR statistic, the following test of hypothesis is performed: H_0 : Cobb-Douglas; H_1 : translog.

Thus, the RL ratio test is determined as follows.

$$LR = -2[\ln LL(H_0) - \ln LL(H_1)] \quad (7)$$

The $LL(H_0)$ refers to the *log-likelihood* value of the estimation using the Cobb-Douglas function, and the $LL(H_1)$ refers to the *log-likelihood* value of estimating the frontier using the translog function. If LR is greater than the value in the Kodde and Palm (1986) table, one rejects H_0 , and if the LR value is inferior to the critical value of the statistic in the Kodde and Palm (1986) table, one does not reject H_0 and the Cobb-Douglas functional form is assumed to be the more appropriate form for the model analyzed (Ferreira and Araújo, 2013).

However, for this work, the functionality test will not be performed, since two frontiers of production will be estimated: a Cobb-Douglas and a translog function. For a translog-type function to be reduced to a Cobb-Douglas function, it is only necessary to impose constant returns of scale to the production function.

Absence of technical progress (TP) test

The maximum likelihood ratio test described in the previous subsection is performed to verify the lack of TP. In this case it is considered whether the coefficients corresponding to the variables related to time (trend) in the production function are equal to zero or not, that is, one tests the null hypothesis that the coefficient referring to time is equal to zero, characterizing the function without the influence

of technological progress, against the alternative hypothesis of the function being estimated considering the influence of the trend term (time). The hypotheses are:

- H_0 : coefficients related to time = 0; and
- H_1 : complete production function.

If the maximum likelihood (ML) value exceeds the critical value in the Kodde and Palm (1986) table, one rejects H_0 and the influence of TP in the function is considered.

3.2.2 Productive efficiency: the Malmquist index

The Malmquist index, formulated by Malmquist (1953), considers the concept of a *distance function* to measure the variations in TFP between two time periods. The distance functions can be specified in relation to sets of inputs or outputs.

The methodology for the analysis was presented originally in Coelli et al. (1998), to obtain the estimation of variation in TFP and to decompose it into its components of change – technical change and technical efficiency.¹²

Coelli et al. (1998), proposed the measurement of the growth of the TFP, being the sum of the variation in the efficiency component and the variation in the technical component. The technology of production, for a given period t , can be defined used the set of production, $P(x_t)$, which represents every output vector, q_t , which can be produced using an input vector x_t :

$$p(x_t) = \{q_t: x_t \text{ can produce } q_t\}. \quad (8)$$

Considering the output-oriented distance function, according to Coelli et al. (1998), the distance function of the output is defined in the production set $P(x)$ as follows.

$$d_0(x, q) = \inf\{\delta: (q|\delta) \in P(x)\}. \quad (9)$$

The distance function $d_0(x, q)$ will have a value which is lesser or equal to one if the output vector q is an element of the feasible production set, $P(x)$. In addition, the distance function will assume value 1 if q is located on the frontier of the feasible set of production, and will assume a value greater than one if q is outside the feasible set of production.

The value of the output-oriented distance function for a state which uses input level x in period t to produce output q_t corresponds to the ratio of the product distance q_t of the i -th state to the frontier of production possibilities, and can be represented as follows.

12. For a more in depth discussion on the Malmquist Index, see Tone and Tsutsui (2017).

$$d_0^t(x_t, q_t) = q_t/F(x_t), \quad (10)$$

in which

$$F(x_t) = \max\{q_t: (x_t, q_t) \in P(x)\}, \quad (11)$$

$F(x_t)$ represents the maximum output that can be obtained given the technology and the level of inputs.

The Malmquist index is defined by the concept of distance functions, which allow the description of a production technology without specifying a behavioral objective function. These distance functions can be defined as input-oriented or output-oriented. The Malmquist TFP index measures the change in TFP between two periods, calculating the distance ratio of each period when compared to a common technology. If the technology of period t is used as reference, the Malmquist TFP index (output-oriented) between periods s and t can be written as follows (Coelli et al., 1998).

$$m_0^t(q_s, x_s, q_t, x_t) = \frac{d_0^t(q_t, x_t)}{d_0^t(q_s, x_s)}. \quad (12)$$

If the technology of period s is used as reference, the index is defined as

$$m_0^s(q_s, x_s, q_t, x_t) = \frac{d_0^s(q_t, x_t)}{d_0^s(q_s, x_s)}, \quad (13)$$

in which:

- $d_0^t(q_t, x_t)$ is the distance function being output-oriented in period t ; and
- $d_0^t(q_s, x_s)$ is the distance function being output-oriented in period s .

A value of m_0 greater than one indicates an increase in TFP from period s to period t , and a value of m_0 which is less than one shows a decrease in TFP between the two periods. These two indices are equivalent only if the technology is Hicks neutral, that is, if the distance function of the output can be represented as:

$$d_0^t(q_t, x_t) = A_t d_0(q_t x_t), \quad (14)$$

for every t . To avoid the imposition of this restriction and the arbitrary choice of one or form or the other to express the index, the Malmquist TFP index is often defined as a geometric mean of these two indices (Coelli et al., 1998).

$$m_0(q_s, x_s, q_t, x_t) = \left[\frac{d_0^s(q_t, x_t)}{d_0^s(q_s, x_s)} \times \frac{d_0^t(q_t, x_t)}{d_0^t(q_s, x_s)} \right]^{1/2}. \quad (15)$$

The distance function in this productivity index can be rearranged to show that it is equivalent to the product of the index of VTE and the index of technical change (or technological change – VT).

$$m_0(q_s, x_s, q_t, x_t) = \frac{d_o^s(q_t, x_t)}{d_o^s(q_s, x_s)} \left[\frac{d_o^s(q_t, x_t)}{d_o^s(q_s, x_s)} \times \frac{d_o^t(q_t, x_t)}{d_o^t(q_s, x_s)} \right]^{1/2}, \quad (16)$$

being:

$$VTE = \frac{d_o^t(q_t, x_t)}{d_o^s(q_s, x_s)}, \quad (17)$$

$$VT = \left[\frac{d_o^s(q_t, x_t)}{d_o^s(q_s, x_s)} \times \frac{d_o^t(q_t, x_t)}{d_o^t(q_s, x_s)} \right]. \quad (18)$$

This shows that the Malmquist total productivity index can be decomposed into the indexes of VTE and variation in technology, and thus one can identify which of these two indices has the greater influence on the variation of TFP (Machado Júnior, Irffi and Benegas, 2011; Araújo, Feitosa and Silva, 2014; Ferreira and Araújo, 2013; Jackson, Johnson and Persico, 2016). However, it should be noted that the Malquimist index has some limitations, such as: it requires a Decision Making Units (DMU) data panel, calculates only the efficiency of a DMU in relation to itself in the past, and requires a support technique (Araújo, Feitosa and Silva, 2014).

In this section, the results obtained through stochastic frontier estimates for the Brazilian states are presented, regarding the education and health sectors after the implementation of the LRF in Brazil. Subsection 4.1 presents the main results in the area of education, while subsection 4.2 features the main results in the health area.

3.3 Education

Table 5 presents the results of the tests applied to verify the Cobb-Douglas functional form when compared to the translog model. The likelihood ratio proves which of the two functions has the best functional form for the model, i.e., one tests the hypothesis in which all the second order coefficients, as well as the coefficients of the cross products of the function, are equal to zero.

It can be observed that, as the value of λ (-15,54) is inferior to the critical value of the statistic in the Kodde and Palm (1986), 26,85, then it cannot be rejected. It can be assumed that the Cobb-Douglas functional form is the most appropriate one for the present analysis. Once the functional form is chosen, the lack of TP was then tested. As already described, the model is estimated in its functional form and in the absence of TP. As can be seen in table 5, the value of

the statistic λ is less than the critical value of the Kodde and Palm (1986) table. Thus, it is not rejected, and the Cobb-Douglas function without TP is chosen as the most appropriate model to be estimated.

TABLE 5
Likelihood ratio test of the parameters of the education frontier of production

Test	Null hypothesis	Value of λ	Critical value	Decision (5% level)
Functional form	Ho: Cobb-Douglas	-15.56	26.85	Reject Ho
Absence of TP	Ho: beta referring to time = 0	-3.42	2.70	Not reject Ho

Authors' elaboration.

As discussed previously, the results of the hypothesis tests indicated as the most appropriate model the Cobb-Douglas production function without TP, according to equation (19). The model was estimated with the variables in logarithm, in order to obtain the value of the coefficients be in terms of elasticities.

$$\ln Txaprov_{i,t} = \alpha_{i,t} + \beta_{1i,t} \ln Gastedu_{i,t} + \beta_{2i,t} \ln Prof_{i,t} + \beta_{3i,t} \ln Escola + v_{i,t} + u_{i,t}. \quad (19)$$

Table 6 presents the results of the stochastic frontier estimation for the education area in Brazilian states, with a Cobb-Douglas functional form, with and without TP. All variables used in the model were statistically significant. Regarding education expenditures per capita, the presence of a coefficient with a positive sign can be observed, indicating that there is a positive relationship between spending on education and the approval rate in secondary education. The value of this coefficient is equal to 0,1279, which indicates that increases in per capita spending on education by 1% increases the approval rate in secondary school by approximately 0.13%.

The number of teachers in the secondary school network variable also showed a positive relationship with the approval rate. The coefficient value of 0,0523 indicates that a 1% increase in the number of teachers in the secondary school system provides an increase in the approval rate of 0.05%. The number of secondary schools variable also showed a positive relationship with the approval rate. The coefficient of this variable is 0,0531, which indicates that a 1% increase in the number of schools causes an increase in the approval rate in secondary education of 0.05%. These results show that public policies aimed at expanding the number of schools and teachers in the secondary school network would, most likely, provide increases in the approval rates in secondary education.

TABLE 6
Results of the stochastic frontier in the area of education (2001-2012)

Variables	Coefficient	Standard deviation	P> z
LnGastedu	0.1279	0.0089	0.000
LnProf	0.0523	0.0067	0.000
LnEscola	0.0531	0.0118	0.000
Constant	3.2837	0.0787	0.000
$\ln \sigma_v^2$	-4.2952	0.5035	0.000
$\ln \sigma_u^2$	1.2485	0.6526	0.056
σ^2	0.0136	0.0068	–
λ	0.7770	0.1130	–
σ_u^2	0.0105	0.0068	–
σ_v^2	0.0030	0.0002	–
Log likelihood = 438.4436	Wald = 293.40	Prob = 0.0000	Number of groups: 27 Number of observations: 324

Authors' elaboration.

These results are not unexpected. It can be said that it is relatively intuitive that greater investment in education, whether increasing the number of teachers or number of schools, would tend to increase the student approval rate.¹³

From the results of the estimation of the model, as well as the application of the previously described methodology, it is possible to decompose the TFP from the Malmquist index, according to table 7. One can see that all Brazilian states are below the technical efficiency frontier (second column of table 7). The state of Tocantins (0,9887) was shown to be the most efficient, followed by the states of Amazonas (0,9627) and Espírito Santo (0,9611). However, the state of Amapá (0,7596) presented the lowest technical efficiency, followed by the states of Rio de Janeiro (0,7623) and Rio Grande do Sul (0,8071).

In relation to TFP, it can be observed that the state of Tocantins (0,9880) was shown to be more productive, followed by the states of Espírito Santo and Amazonas. However, the state of Amapá was the least productive.

These results mostly follow what has been found previously in works which analyze Brazil, such as Brunet et al. (2006) and Machado Júnior, Irffi and Benegas (2011), i.e., it seems to be the norm in the country that states and municipalities in Brazil are not technically efficient in terms of education spending.

13. For a comprehensive literature review on the subject, we recommend the work of Jackson, Johnson and Persico (2016).

TABLE 7
Decomposition of the TFP for each state, measured by the Malmquist index

States	Technical efficiency	Technological var.	TFP	TFP ranking
RO	0.9536	0.9911	0.9451	6
AC	0.9326	0.9783	0.9124	12
AM	0.9627	0.9986	0.9614	3
RR	0.8674	0.9654	0.8374	23
PA	0.9417	1.0200	0.9605	4
AP	0.7596	0.9689	0.7360	27
TO	0.9887	0.9993	0.9880	1
MA	0.8632	1.0346	0.8931	14
PI	0.8643	1.0047	0.8684	18
CE	0.9450	1.0141	0.9583	5
RN	0.8215	0.9937	0.8163	25
PB	0.9259	1.0113	0.9364	9
PE	0.9406	0.9972	0.9380	8
AL	0.9145	1.0130	0.9264	10
SE	0.8623	1.0251	0.8839	16
BA	0.8550	1.0207	0.8727	17
MG	0.8942	1.0274	0.9187	11
ES	0.9611	1.0014	0.9624	2
RJ	0.7623	1.0299	0.7851	26
SP	0.8185	1.0346	0.8468	21
PR	0.8889	1.0148	0.9021	13
SC	0.8883	0.9542	0.8476	19
RS	0.8071	1.0188	0.8222	24
MS	0.8858	0.9986	0.8846	15
MT	0.8493	0.9973	0.8470	20
GO	0.9313	1.0124	0.9428	7
DF	0.8460	0.9994	0.8455	22

Authors' elaboration.

3.4 Health

Table 8 presents the results of the tests applied to verify the Cobb-Douglas functional form compared to the translog model for the health area. It can be observed that since the value of λ (-37,29) is less than the critical value of the statistics in the Kodde and Palm (1986) table, of 21,26, it cannot be rejected. It can be assumed that the most appropriate model for this work is using the Cobb-Douglas functional form. Thus, the model is estimated using that functional form and considering the absence of TP.

TABLE 8
Likelihood ratio test of the parameters of the health frontier of production

Test	Null hypothesis	Value of λ	Critical value	Decision (5% level)
Functional form	Ho: Cobb-Douglas	-37.29	21.26	Not reject Ho
Absence of TP	Ho: beta referente ao tempo = 0	-25.15	2.70	Not reject Ho

Authors' elaboration.

Thus, the model was estimated with the variables in logarithm according to equation (20), in order to obtain the value of the coefficients be in terms of elasticities.

$$\ln Txmort_{i,t} = \alpha_{i,t} + \beta_{1i,t} \ln Gastsaud_{i,t} + \beta_{2i,t} \ln Leitos_{i,t} + \beta_{3i,t} \ln Enfi_{i,t} + \beta_{4i,t} \ln Med_{i,t} + \beta_{5i,t} \ln Txcobert_{i,t} + V_{i,t} + u_{i,t}. \quad (20)$$

Table 9 presents the results of the estimation of the stochastic frontier of the Brazilian states for the health area, with the model in the Cobb-Douglas functional form, with and without TP. All variables used in the model were statistically significant, except for the number of hospital beds variable.

Regarding health expenditures per capita, one can observe the coefficient with a negative sign, indicating that there is a negative relationship between spending on health and the infant mortality rate. This implies that increases in health spending would lead to reductions in infant mortality rates in Brazilian states. The value of this coefficient is equal to -0.2521, which indicates that increases in health expenditures per capita by 1% would reduce the infant mortality rate by 0.25%.

The number of nurses variable also presented a negative relationship with the infant mortality rate. The coefficient value -0.0847 indicates that a 1% increase in the number of nurses hired provides a reduction in the infant mortality rate of 0.08%. The number of physicians variable also had a negative relationship with the infant mortality rate. The coefficient of this variable was -0.0323, indicating that a 1% increase in the number of doctors hired would cause a reduction in the infant mortality rate of 0.03%. These results suggest that increases in the hiring of health professionals by the state would lead to reductions in infant mortality rates.

The PSF coverage rate variable presented a coefficient equal to -0.0694, indicating that an increase in the coverage rate of the PSF by 1% would lead to a reduction in the infant mortality rate of approximately 0.07%. In other words, public policies that aim to expand the PSF program coverage may achieve satisfactory results in health indicators, such as decreasing the infant mortality rate.

All the previous results seem quite intuitive, in the sense that a higher investment in health tends to reduce mortality indicators, and has a broad empirical basis in the literature.¹⁴

TABLE 9
Results of the stochastic frontier in the health area (2001-2012)

Variables	Coefficient	Standard deviation	P> z
<i>LnGastsaud</i>	-0.2521	0.0191	0.000
<i>LnLeitos</i>	-0.0049	0.0391	0.898
<i>LnEnf</i>	-0.0847	0.0150	0.000
<i>LnMed</i>	-0.0323	0.0167	0.054
<i>LnTxcobert</i>	-0.0694	0.0251	0.006
<i>Constant</i>	4.4371	0.1226	0.000
$\ln \sigma_v^2$	-2.3529	0.3782	0.000
$\ln \sigma_u^2$	2.4506	0.4213	0.056
σ^2	0.0951	0.0359	–
λ	0.9206	0.0308	–
σ_u^2	0.0875	0.0359	–
σ_v^2	0.0075	0.0006	–
<i>Log likelihood = 269.13843</i>	Wald = 1129.51	Prob = 0.0000	Number of groups: 27 Number of observations: 324

Authors' elaboration.

Table 10 shows the decomposition of the TFP from the Malmquist index for the health area. It can be observed that, just as in the area of education, all the Brazilian states are featured below the technical efficiency frontier. The Brazilian state with the most technical efficiency is Tocantins (0.8663), while the state of Paraná (0.3561) had the lowest technical efficiency.

In relation to the TFP, it can be verified that the state of Tocantins (0.8313), as was the case in education, was more productive, followed by the states of Mato Grosso (0.8205) and Acre (0.7808). On the other hand, the state of Paraná (0.3543) was shown to be the least productive.

14. See, for example, the compilation of articles by the IMF (2004) and Maruthappu, Hasan and Zeltner (2015) for a thorough discussion on the subject.

TABLE 10
Decomposition of the TFP for each state, measured by the Malmquist index

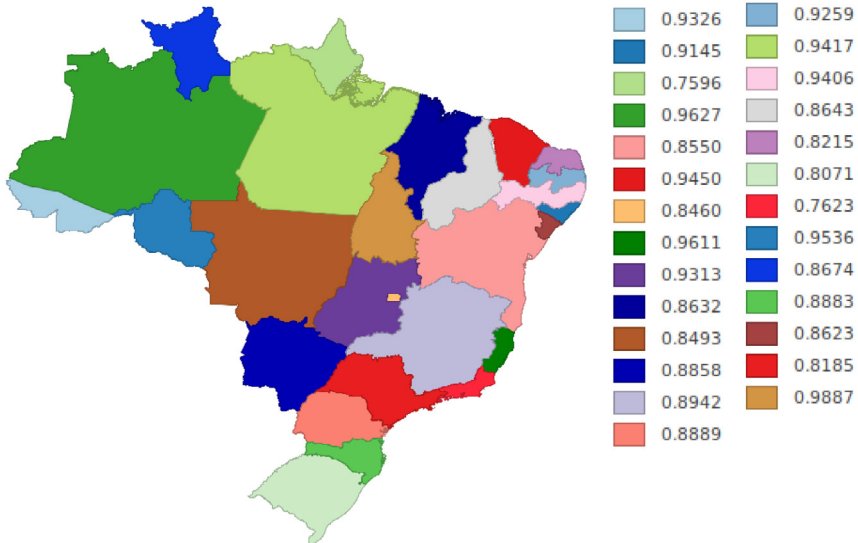
States	Technical efficiency	Technological var.	TFP	TFP ranking
RO	0.7146	0.9898	0.7074	7
AC	0.7934	0.9841	0.7808	3
AM	0.7027	0.9999	0.7027	8
RR	0.7444	0.9704	0.7224	6
PA	0.6900	0.9957	0.6871	9
AP	0.7685	0.9947	0.7645	4
TO	0.8663	0.9595	0.8313	1
MA	0.5841	0.9261	0.5409	22
PI	0.6439	0.9079	0.5847	16
CE	0.5716	0.9344	0.5341	23
RN	0.6869	0.9406	0.6462	12
PB	0.6535	0.9557	0.6246	14
PE	0.6434	0.9451	0.6081	15
AL	0.6399	0.9023	0.5773	17
SE	0.6945	0.9805	0.6810	10
BA	0.6752	1.0912	0.7368	5
MG	0.5286	1.0806	0.5713	18
ES	0.4493	0.9568	0.4299	26
RJ	0.5138	1.0982	0.5643	19
SP	0.4717	1.0950	0.5166	24
PR	0.3561	0.9946	0.3543	27
SC	0.5647	0.9893	0.5587	20
RS	0.6724	0.9630	0.6476	11
MS	0.5524	1.0044	0.5549	21
MT	0.6699	1.2247	0.8205	2
GO	0.4984	0.9890	0.4929	25
DF	0.6226	1.0273	0.6396	13

Authors' elaboration.

Again, these results are in line with previous works, such as Brunet et al. (2006), particularly considering the result that states which spend more, such as in the Southeast region of Brazil (São Paulo and Rio de Janeiro) appear to be less efficient than other regions of the country, such as the Center-West, and Barbosa and Souza (2015) and Mazon, Mascarenhas and Dallabrida (2015), which discuss the lack of efficiency in providing public goods such as health in vastly different places, such as the Northeast region and the Brazilian South.

Furthermore, featured below are four figures which highlight, in visual terms, the different levels of technical efficiency of the Brazilian states, using different colors, in order to facilitate comprehension of the previous discussion.

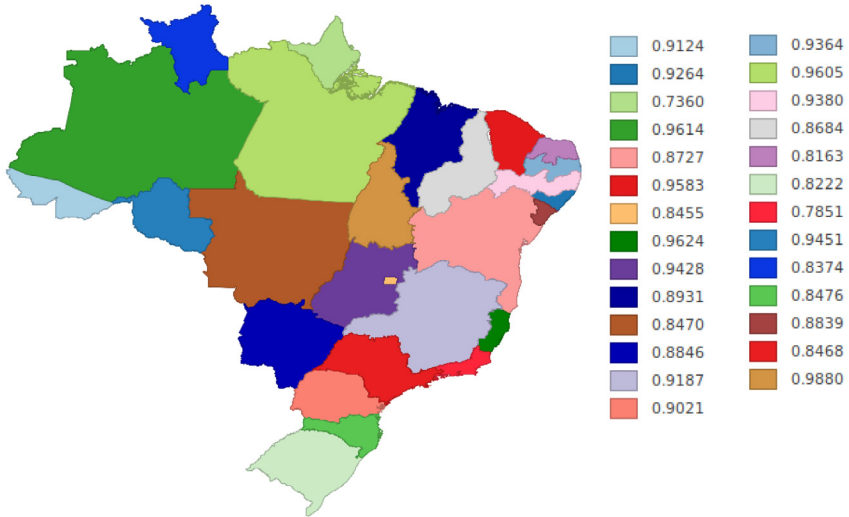
FIGURE 1
Technical efficiency of education spending by federative units – Brazil



Authors' elaboration.

Publisher's note: Figure whose layout and texts could not be formatted and proofread due to the technical characteristics of the original files.

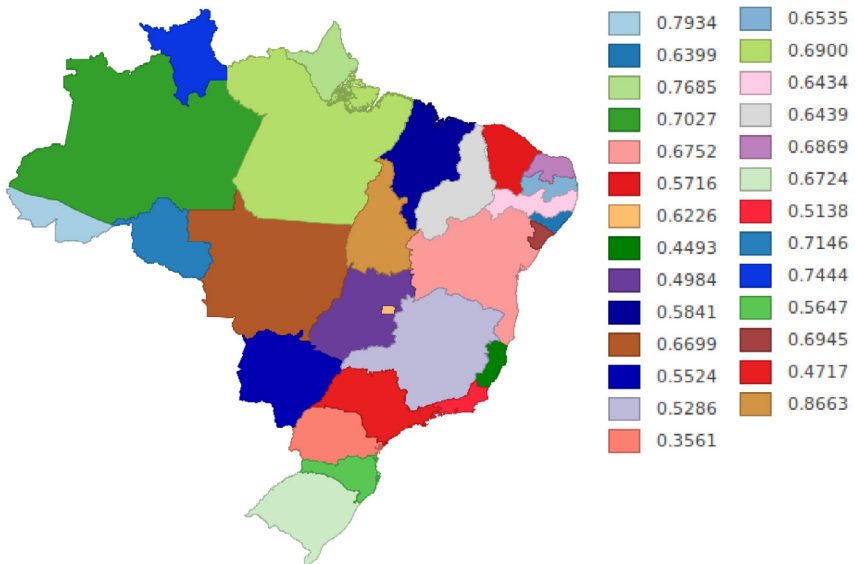
FIGURE 2
TFP of education spending by federative units – Brazil



Authors' elaboration.

Publisher's note: Figure whose layout and texts could not be formatted and proofread due to the technical characteristics of the original files.

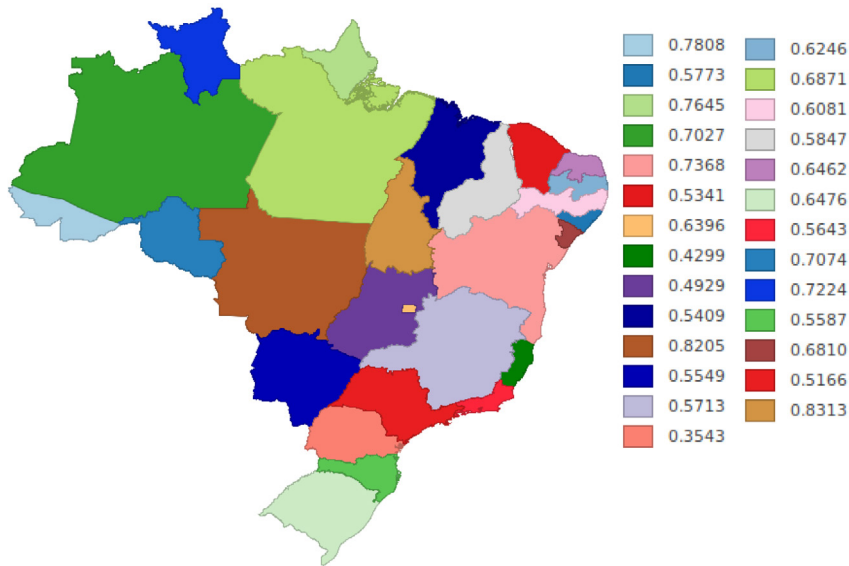
FIGURE 3
Technical efficiency of health spending by federative units – Brazil



Authors' elaboration.

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FIGURE 4
TFP of the health expenses of the federative units – Brazil



Authors' elaboration.

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4 CONCLUDING REMARKS

The objective of this work was to verify the technical efficiency of the Brazilian states in relation to public spending on health and education after the implementation of the LRF. For this, a parametric model for the estimation of the stochastic frontier of production and the decomposition of the VTE and technological variation by means of the Malmquist index for the Brazilian states were used.

The results showed that, in relation to expenditures in education, there is a positive and statistically significant relationship between spending on education per capita and the approval rate in secondary education. Other variables showed an influence on the approval rate, such as the number of schools and teachers in the secondary education network. This suggests that public policies aimed at expanding the number of schools and teachers are important for raising the approval rate.

Regarding the decomposition of the VTE and technological variation by means of the Malmquist index in education, all the Brazilian states were shown to be below the frontier of technical efficiency. The state of Tocantins proved to be the most efficient and with the highest productivity index in TFP. A surprising fact was that the states of Amapá and Rio de Janeiro were shown to be well below

the frontier of technical efficiency and presented low indexes of productivity in the TFP. That is, Brazilian states are not being efficient considering expenditures on education, however Amapá and Rio de Janeiro are particularly in a more worrisome situation. One possible solution would be to review those investments to optimize spending on schools and increase the hiring of teachers.

Regarding the health area, the results show that health expenditures per capita have a negative relationship with the infant mortality rate in the Brazilian states. That is, increases in spending lead to reductions in the infant mortality rate. Other variables were found to be important in the fight against child mortality, such as the increase in the hiring of health professionals such as doctors and nurses and the increase in the PSF coverage rate.

In relation to the decomposition of the VTE and technological variation through the Malmquist index in health, as was the case in education, all the Brazilian states were shown to be below the frontier of technical efficiency. In this area, one can also highlight the state of Tocantins, which has proved to be the most efficient and with the highest productivity index in TFP.

Overall, one can conclude that, according to the results obtained, there is some indication that the Brazilian states are not being efficient in their spending on education and health. It is suggested, therefore, that public policies focused on education should invest more in infrastructure and human capital of the teaching staff in the secondary education network; as well as policies in the health area should invest in hiring health professionals; also, it would be advisable to expand the coverage of the PSF.

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