INFLATION EXPECTATIONS AND THE PHILLIPS CURVE: AN ENCOMPASSING FRAMEWORK[1]

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This paper contrasts empirically four leading models of inflation dynamics – the Accelerationist Phillips curve (APC), New Keynesian Phillips curve (NKPC), Hybrid Phillips curve (HPC) and Sticky Information Phillips curve (SIPC). We employ an encompassing Phillips curve specification that allows us to derive tests for these models within a single framework. Using the generalized method of moments (GMM) estimator, the evidence suggests that the restrictions implied by the NKPC, HPC, and SIPC are rejected for the period after the Real Plan in Brazil. Only the restrictions implied by the APC are not rejected. However, when we construct confidence regions that are robust to weak instruments, it is not possible to reject any of the Phillips curve specifications, including the NKPC.

Keywords: Phillips curves; weak instruments; fully robust confidence regions.

EXPECTATIVAS DE INFLAÇÃO E A CURVA DE PHILLIPS: UM ARCABOUÇO ABRANGENTE

Este artigo contrasta empiricamente quatro importantes modelos de dinâmica da inflação – a curva de Phillips Aceleracionista (APC), a curva de Phillips Novo Keynesiana (NKPC), a curva de Phillips Híbrida (HPC) e curva de Phillips de Informação Rígida (SIPC). Utilizamos uma especificação abrangente da curva de Phillips que nos permite testar esses modelos dentro de um mesmo arcabouço. Utilizando estimador do método generalizado dos momentos (GMM), a evidência sugere que as restrições decorrentes da NKPC, da HPC e da SIPC são rejeitadas para o período após o Plano Real no Brasil. Apenas as restrições decorrentes da APC não são rejeitadas. Porém, quando construímos regiões de confiança que são robustas ao problema de instrumentos fracos, não é possível rejeitar nenhuma das especificações da curva de Phillips, incluindo a NKPC.

Palavras-chave: curvas de Phillips; instrumentos fracos; regiões de confiança totalmente robustas.

JEL: E12; E31; E32.

1 INTRODUCTION

The Phillips curve has been playing a central role in policymakers' understanding of the macroeconomy and in the formulation of monetary policy. It is not surprising then that empirical challenges in estimating a Phillips curve relationship have been closely intertwined with challenges in conducting monetary policy. Much work has been done, both theoretically and empirically, since Phillips's seminal 1958

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paper. Yet economists have not converged to a widely agreed specification that is satisfactory both from a theoretical and an empirical standpoint.

The empirical evidence shows that inflation tends to be pro-cyclical: periods of above average inflation tend to be associated with above average economic activity. This statistical relationship is known as the Phillips curve. The Phillips curve was perceived in the 1960's as a menu for monetary policymakers: they could choose between high inflation and low unemployment or low inflation and high unemployment. But this interpretation of the Phillips curve assumed that the relationship between unemployment and inflation was stable and would not break down when a policymaker attempts to exploit the tradeoff. After Friedman's (1968) paper and the high inflation episodes experienced by many economies in the 1970s, this interpretation of the Phillips curve was discredited. After a period of low inflation in the 1980s and early 1990s, economists again worked on a theoretical framework for the Phillips curve. The New Keynesian Phillips curve (NKPC) provides an interpretation of the short-run inflation-unemployment trade-off by deriving it from an optimizing framework featuring rational expectations and nominal rigidities. This is a structural model, designed to be capable of explaining the behavior of inflation without being subject to the Lucas critique. The NKPC is part of the New Keynesian model which is the workhorse model for monetary analysis. However, to use the NKPC for policy analysis requires it to have a good econometric track record in describing inflation dynamics.

Initial attempts to estimate the NKPC using aggregate time series data for the U.S. were not very successful (Galí and Gertler, 1999): the estimated coefficient on the output gap (proxied by detrended real GDP) was small and often negative in quarterly data. One interpretation for the poor results using a standard output gap measure is that it is a poor proxy for real marginal cost, which according to the theory, would be the appropriate variable. Using the generalized method of moments (GMM) Galí and Gertler report evidence in favor of the New Keynesian Phillips curve when labor's share of income, rather than a standard output gap variable, is used to proxy for real marginal cost. In order to capture the inflation persistence found in the data, Galí and Gertler modify the basic Calvo model of sticky prices to introduce lagged inflation into the Phillips curve, called hybrid Phillips curve. Based on U.S. data and using real marginal cost as the forcing variable, Galí and Gertler conclude that not only the forward-looking behavior is predominant but, given the small estimate of the degree of backwardness, the pure forward-looking model may do a reasonably good job of describing the data. Galí, Gertler and López-Salido (2001) provided evidence on the fit of the NKPC for the Euro area.

As an alternative to the models of sticky prices, Mankiw and Reis (2002) argue that sticky information – the slow dispersion of information about macroeconomic conditions – can help account for the sluggish adjustment of prices and for the real effects that occur in response to monetary shocks. Kiley (2007) attempted to test the sticky information model of inflation against the sticky price for the United States using maximum-likelihood techniques. He finds that, once hybrid behavior is allowed for, hybrid sticky-price models provide a better description of inflation dynamics than a sticky-information model.

Following the steps of Galí and Gertler, several authors have attempted to estimate the hybrid Phillips curve for Brazil employing the GMM to test the empirical fit of the NKPC for Brazil, giving special attention to the relative importance of the backward and forward-looking components of inflation.[4] Areosa and Medeiros (2007) derive and estimate a structural model for inflation in a small open economy. They find the following conclusions:

- the backward-looking component has smaller values than that of a closed economy, between 0.10 and 0.37;
- the forward-looking component shows higher values than that of a closed economy, between 0.63 and 0.81; and
- the impact of the marginal cost, despite being a little irrelevant, is statistically significant.

Mazali and Divino (2010) apply for Brazil the new Keynesian model of Blanchard and Galí (2007) with real wage rigidity and supply shocks. As the estimated coefficients satisfied a set of restrictions imposed by the theoretical model and over-identifying restrictions were not rejected, they concluded that the estimated NKPC adjusted very well to the Brazilian data. Their point estimate of the coefficient of lagged inflation is 0.59 and that of expected inflation is 0.41. Their results showed a negative and statistically significant relationship between inflation and unemployment. Mendonça, Sachsida and Medrano (2012) also based on the specification of Blanchard and Galí, but employing different proxies for inflation and unemployment concluded that the NKPC has difficulty fitting inflation dynamics in Brazil. Medeiros, Portugal and Aragón (2017) based on the hybrid specification of Galí and Gertler concluded that the forward-looking component of inflation is dominant, though its importance has been reduced after 2004. As we will see next, weak instrument issues provide a unifying explanation of the sensitivity of NKPC estimates and of the puzzling disagreement between analyses based on standard inference procedures.

As the popularity and usage of the NKPC has grown, criticisms have been raised with respect to its empirical identifiability. The main issue is that instrumental variables (IV) methods such as GMM are not immune to the presence of weak

4. For a survey on the studies about the Phillips curve in the Brazilian economy, see Sachsida (2013).

instruments.[5] Identification difficulties have led to re-examinations of NKPC models, and in particular of the Gali and Gertler NKPC specification. Several authors have argued that Gali and Gertler's results are unreliable because the parameters of the hybrid NKPC are weakly identified and they are estimated using methods that are not robust to identification problems (also known as weak instruments).[6] The weak instruments literature has shown that using conventional inference methods after pretesting for identification is both unreliable and unnecessary. A better approach is to construct confidence regions that are fully robust to weak instruments.[7]

This paper contrasts empirically four leading models of inflation dynamics – the Accelerationist Phillips curve (APC), New Keynesian Phillips curve (NKPC), Hybrid Phillips curve (HPC), and Sticky Information Phillips curve (SIPC). Given that we estimate reduced-form models, the choice of the sample is motivated by the attempt of avoiding the Lucas critique by selecting periods of economic regime stability. There are two main contributions of this paper to the empirical literature that analyses inflation dynamics in Brazil. First, our method of testing Phillips curves is different from the approaches taken by previous studies because it is based on an alternative specification of this curve that encompasses the APC, NKPC, HPC and SIPC. This encompassing specification has the advantage of reducing part of the vast specification uncertainty surrounding the Phillips curve by making it possible to test each of these alternative specifications within a single framework. Second, in order to conduct inference on the parameters of the empirical model we use methods that are robust to the weak instruments problem that arises when the GMM is employed to estimate the Phillips curve. To the best of our knowledge, none of the studies that employed GMM to estimate the NKPC for Brazil employed identification robust methods, making their results unreliable according to the weak instruments literature.

Using conventional GMM, our preliminary results suggest that the NKPC, HPC and SIPC are not consistent with data for Brazil after the Real Plan. Only the APC is consistent with these data. However, when we construct confidence regions that are robust to weak instruments in the sense that identification of the coefficients is not assumed (in contrast to the conventional GMM method, where the validity of tests of estimated coefficients requires the assumption that they are identified), our previous conclusions turn on their head and making it impossible to reject any of the Phillips curve specifications. This happens because conventional GMM confidence regions understate the sampling uncertainty, compared to regions

5. Weak instruments arise when the instruments in linear IV regression are weakly correlated with the included endogenous variables. In GMM, more generally, weak instruments correspond to weak identification of some or all of the unknown parameters. If instruments are weak, then the sampling distributions of GMM and IV statistics are in general nonnormal, and standard GMM and IV point estimates, hypothesis tests, and confidence intervals are unreliable.

6. See Ma (2002), Dufour, Khalaf and Kichian (2006), Nason and Smith (2008), Kleiberger and Mavroeidis (2009).

7. See, for example, Stock, Wright and Yogo (2002), Dufour (2003), Andrews and Stock (2005).

that are robust to weak instruments. The results do not depend on the choice of the forcing variable (output gap or marginal cost) in the Phillips curve equation.

The paper is organized as follows. Section 2 reviews briefly the history of the Phillips curve. Section 3 presents the encompassing Phillips curve (EPC) and shows how different Phillips curve specifications considered in the literature can be seen as special cases of the EPC. Section 4 extends the EPC to the open economy. Section 5 tests for Brazil the restrictions implied by different Phillips curve specifications on a quarterly sample from 1996 to 2015. Section 6 discusses the issue of weak instruments and presents two approaches used to deal with it. Finally, section 7 brings some concluding remarks.

2 AN OVERVIEW OF THE PHILLIPS CURVE

In a seminal paper, Phillips (1958) showed that there was a negative and relatively stable relationship between nominal wage inflation and unemployment in the United Kingdom over the previous century. This relationship was found to work well for price inflation and for other economies, receiving the name of Phillips curve. It became a key part of the standard Keynesian textbook model of the 1960s and as Keynesian economists saw it, the Phillips curve provided an exploitable trade-off between inflation and unemployment: policymakers could use demand management policies to increase output and decrease unemployment, but this could only be done at the expense of higher inflation. The Phillips curve relationship can be represented as

$$\pi_t = \alpha - \gamma u_t,$$

in which π_t is inflation, u_t is the unemployment rate, and $\gamma > 0$.

The theoretical foundations of these early formulations were not completely sound, with a particular weak point being their treatment of how expectations entered wage and price setting. This weakness was thoroughly criticized in the seminal contributions of Phelps (1967; 1968) and Friedman (1968). Friedman predicted that attempts to keep unemployment low at the expense of higher inflation would just result in higher inflation expectations. Thus, the economy would not be able to sustain the low unemployment and would end up with higher inflation. In the Friedman-Phelps framework, then, there is no permanent trade-off between the level of inflation and the unemployment rate. However, to the extent that agents' expectations were slow to catch up with reality, a policymaker could keep unemployment below the natural rate by constantly boosting the inflation rate. For this reason, the Friedman-Phelps characterization of the inflation process also came to be known as the "accelerationist hypothesis" since an acceleration in prices would occur should policymakers attempt to permanently keep unemployment below its natural rate. Phelps assumed that inflation expectations evolved over time as a result of actual past experience – that is, that expectations were formed adaptively.[8]

Friedman argued that the correct formulation of the inflation-unemployment tradeoff was a Phillips curve of the form:

$$\pi_t = E_{t-1}\,\pi_t - \gamma(u_t - \bar{u}),$$

in which inflation, π_t, is negatively correlated with deviations of the unemployment rate from its natural rate $\bar{u}$ ($\gamma > 0$) and where the entire curve is shifted up or down one-for-one with changes in the rate of inflation that agents expected at time t - 1 to prevail at time t, $E_{t-1}\,\pi_t$. A common variant of this equation replaces $u_t - \bar{u}$ with the gap between actual and potential output, $y_t - \bar{y} \equiv x_t$. There is a long tradition in applied work that assumes backward-looking expectations: expected inflation is determined by past inflation. In the special case in which $E_{t-1}\,\pi_t = \pi_{t-1}$, the Phillips curve becomes

$$\pi_t = \pi_{t-1} - \gamma(u_t - \bar{u}).$$

This so-called accelerationist Phillips – curve in which the acceleration of prices is related to unemployment – embodied two critical innovations in the literature. First, it eliminated the long-run trade-off between inflation and unemployment that was inherent in the original Phillips curve model. Second, it began to emphasize the importance of expectations in the price-setting process, a change that was to have dramatic implications on the evolution of inflation models.

In the decade following the publication of the Phelps and Friedman papers, the notion that the accelerationist view of the inflation process was correct gained wider acceptance. Several factors contributed to this attitude. The first, of course, was the strength of the theoretical arguments themselves. Second, it became apparent by the mid-1970s that the inflation-unemployment tradeoff implied by the short-run Phillips curve had shifted. Finally, it became easier to find that the lags of inflation in empirical Phillips curves summed to one. In addition, the important contribution of "supply shocks" to price acceleration in the early 1970s led to food, energy, and/or import prices receiving special treatment in empirical descriptions of inflation. What emerged in this period, therefore, was a benchmark econometric model of inflation of the form:

$$\pi_t = \alpha - \gamma u_t + B(L)\pi_{t-1} + \lambda z_t + \varepsilon_t,$$

in which $B(L)$ is the distributed lag operator with $B(1) = 1$, z_t denotes a vector of supply shocks, and ε_t is an error term. In this specification, then, inflation dynamics are determined by three sources: real activity (as summarized by the unemployment rate), supply shocks, and "inertia" (as captured by the lagged inflation terms). For this reason, it is sometimes called the "triangle model".

8. In Phelps (1967), the appeal to adaptive expectations is explicit. The term is not used by Friedman (1968), who provides an informal discussion of a gradual adjustment process.

Taken literally, the characterization of inflation dynamics that the triangle model provides carries important implications for the conduct of macroeconomic policy. To the extent that lagged inflation captures true inertia in the price-setting process (resulting, for instance, from how expectations are formulated), the model implies that rapid reductions in inflation can only be produced at the cost of a substantial increase in unemployment. Hence, the model points to a gradualist approach as providing the best way to effect a large reduction in inflation. In addition, policymakers must be mindful of the presence of long time lags between macroeconomic shocks (including policy actions) and their full effects on inflation. Thus, this framework provides a strong argument in support of preemptive action to head off the full effect of an inflationary shock.

The introduction of rational expectations into the modeling of economic dynamics had a significant influence on the development of macroeconomic theory from the mid-1970s onwards. The "demise" of the traditional Phillips curve, and the sense that it was due to inadequate modeling of expectations, was a major impetus for the rational expectations school led by Robert Lucas and Thomas Sargent. Lucas and Sargent also rejected the "accelerationist" reformulation of the Phillips curve because it relied on the assumption of adaptive expectations, which do not allow for the idea that agents process information in an optimal manner. In addition to being more precise about expectations formation, this school of economists relied more heavily on neoclassical "microfoundations" for macroeconomic models. Often, as well as rejecting the Phillips curve, these economists also questioned the whole basis for Keynesian economics, i.e. the assumption that monetary policy could systematically affect output even in the short-run.

The principal response of Keynesian economists to these theoretical critiques has been to attempt to build models that incorporate rational expectations and that provide a microeconomic justification for monetary policy having, at least, short-run effects. To explain why monetary policy might have effects on the economy, one needs a theory of why inflation is not just determined by some nominal anchor such as the money supply. The most common microeconomic rationale put forward has been sticky prices. With sticky prices, an increase in the money stock can produce a short-run increase in real spending power and thus can boost real output. Many academic economists have become convinced that certain theoretical new Keynesian models can provide a good description of the empirical inflation process. In part, this development stemmed from the realization that a number of popular new Keynesian models of price-setting each implied a sort of Phillips curve relationship, known as the NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t,$$

in which x_t is a measure of output gap.

In these models inflation is determined in a completely forward-looking manner. The idea that there is considerable inertia in inflation and hence that it is difficult to reduce inflation quickly, does not hold in this framework – indeed, according to the NKPC, there is no "intrinsic" inertia in inflation, in the sense that there is no structural dependence of inflation on its own lagged values. Thus, the NKPC has very different implications for monetary policy. This model implies that there is no need for gradualist policies to reduce inflation. According to the NKPC, low inflation can be achieved immediately by the central bank announcing (and the public believing) that it is committing itself to eliminating positive output gaps in the future.

Many estimates of the NKPC find that lagged inflation helps to explain current inflation. Galí and Gertler (1999) consider augmenting the NKPC with a backward-looking element that is motivated by the presence of some firms that follow a simple rule of thumb in setting prices. Christiano, Eichenbaum and Evans (2005) derive a similar specification under the assumption that price-setters who are unable to reset prices instead index their prices to the last period inflation rate. All of these variants imply a so-called HPC of the form

$$\pi_t = \gamma_b \pi_{t-1} + \lambda_f E_t \pi_{t+1} + \kappa x_t.$$

The model of Mankiw and Reis (2002) was pioneer in the literature on sticky information. According to it, a Phillips curve with this rigidity is an adequate representation of the structural relationship between inflation and the real side of the economy. The model assumes that acquiring information is costly, and as a result information about macroeconomic conditions diffuses slowly through the population. Specifically, Mankiw and Reis assume that in each period a fraction of firms acquires complete (perfect) information about the current state of the economy, and these firm set prices optimally based on this information. The remaining firms continue to set prices based on outdated information. Mankiw and Reis posit that what matters now for current inflation is not current expectations about future economic conditions, but past expectations about current economic conditions. Because information constraints can apply to all economic agents, the sticky information model potentially provides a unifying framework for explaining the inertial behavior of different macroeconomic variables.

3 PHILLIPS CURVE: AN ENCOMPASSING SPECIFICATION

The Encompassing Phillips Curve (EPC) – a model of inflation dynamics that encompasses the NKPC, APC, HPC and SIPC as special cases – takes the following form:

$$\Delta\pi_t = \alpha_1 \Delta\pi_{t-1} + \alpha_2 \Delta\pi_{t-2} + \beta_0 \Delta x_t + \gamma_1 \Delta x_{t-1} + \delta_1 \pi_{t-1} + \varepsilon_t, \tag{1}$$

in which π is the inflation rate, x is a measure of inflation pressure (usually the output gap or, alternatively, the marginal cost), $\Delta z_t \equiv z_t - z_{t-1}$ is the rate of change of variable z and ε_t is an error term that can be correlated with the explanatory variables. This encompassing specification implies that the change of inflation depends on its lagged values, the change of the output gap (marginal cost), the lagged level of output gap (marginal cost) and the lagged level of the inflation rate. If the coefficient of this last variable is different from zero there is a long-run level trade-off between inflation and output gap (marginal cost).[9]

Let us show how each model is embedded in equation (1). For the case in which expected inflation depends on past inflation (we assume that expected inflation is the average of the last three periods), the APC is given by

$$\pi_t = \omega_1\pi_{t-1} + \omega_2\pi_{t-2} + \omega_3\pi_{t-3} + \kappa_0 x_t + \kappa_1 x_{t-1} + \varepsilon_t, \sum_{i=1}^{3} \omega i = 1\,, \kappa_0 + \kappa_1 > 0.$$

This equation can be rewritten as

$$\Delta\pi_t = -(1-\omega_1)\Delta\pi_{t-1} - (1-\omega_1-\omega_2\Delta\pi_{t-2}) + \kappa_0\Delta x_t + \kappa_1\Delta x_{t-1} + \varepsilon_t\,.$$

This specification is a particular case of equation (1) when:

$$\alpha_1 = -(1-\omega_1) < 0, \alpha_2 = -(1-\omega_1-\omega_2) < 0, \beta_0 = \kappa_0 > 0, \gamma_1 = \kappa_1 > 0, \delta_1 = 0.$$

The NKPC can be expressed as

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + \varepsilon_t,$$

in which the rate of inflation depends on the rate of inflation that agents expect to prevail at time t+1, with the information available at time t. Assuming rational expectations,

$$E_t\pi_{t+1} = \pi_{t+1} + \mu_{t+1}\,,$$

in which μ_{t+1} is a white noise error term, the NKPC can be written as

$$\Delta\pi_t = -(\kappa/\beta)x_{t-1} + [(1-\beta)/\beta]\pi_{t-1} + \eta.$$

The stochastic term η is given by: $\eta \equiv \mu_t - \varepsilon_{t-1}/\beta$, which is correlated with lagged inflation. This is a particular case of equation (1) when:

$$\alpha_1 = \alpha_2 = \beta_0 = 0, \gamma_1 = -\kappa/\beta < 0, \delta_1 = (1-\beta)/\beta > 0.$$

9. Rudd and Whelan (2006) reports reduced form regressions for $\Delta\pi$_{t} using the specification of equation (1). They have found negative coefficients on the lagged changes in inflation and positive coefficients for output gap and labor share. The coefficient of labor share is not significant. The main goal of Rudd and Whelan (2006) paper was to assess the empirical performance of HPC for the US inflation. They did not propose using equation (1) as a framework to test all Phillips curves specifications, which is our goal in this paper.

The HPC specification assumes that the inflation rate depends on the past inflation rate, the expected inflation rate for the next period and an inflation pressure variable according to:[10]

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \kappa x_t + \varepsilon_t.$$

We assume rational expectations, as with the NKPC, and after some algebra we obtain

$$\Delta\pi_t = \frac{\gamma_b}{\gamma_f}\Delta\pi_{t-1} - \frac{\kappa}{\gamma_f}x_{t-1} + \frac{(1-\gamma_b-\gamma_f)}{\gamma_f}\pi_{t-1} + \xi,$$

in which $\xi \equiv \mu_t - \varepsilon_{t-1}/\gamma_f$, which is correlated with both lagged inflation and lagged change of inflation. This is a particular case of equation (1) when

$$\alpha_1 = \gamma_b/\gamma_f > 0, \alpha_2 = \beta_0 = 0, \gamma_1 = -\kappa/\gamma_f < 0, \delta_1 = (1-\gamma_b-\gamma_f)/\gamma_f \geq 0.$$

The SIPC derived by Mankiw and Reis (2002) is given by[11]

$$\pi_t = \sum_{j=0}^{\infty}(1-\lambda)^j E_{t-1-j}(\pi_t + \alpha\Delta x_t) + \frac{\alpha\lambda}{(1-\lambda)}x_t + \varepsilon_t.$$

Inflation depends on the current output gap and on a geometric sum of past expectations of current inflation and output growth relative to potential. Assuming rational expectations and using the lag operator $L^j E_{t-1} = E_{t-1-j}$ we obtain, after some algebra, the following expression for the acceleration of inflation

$$\Delta\pi_t = \frac{\alpha\lambda(2-\lambda)}{(1-\lambda)^2}\Delta x_t + \frac{\alpha\lambda^2}{(1-\lambda)^2}x_{t-1} + \varsigma,$$

in which $\varsigma \equiv [1/(1-\lambda)][\varepsilon_t - (1-\lambda)\varepsilon_{t-1} + (\pi_t - E_t\, \pi_t)]$. Thus, the change of inflation depends on the change of output gap (marginal cost) and its lagged level. This expression is a particular case of equation (1) when

$$\alpha_1 = \alpha_2 = \delta_1 = 0, \beta_0 = \alpha\lambda(2-\lambda)/(1-\lambda)^2 > 0, \gamma_1 = \alpha\lambda^2/(1-\lambda) > 0.$$

The EPC provides a simple set-up to test competing specifications of the Phillips curve. Table 1 shows the signs of the coefficients of the EPC resulting from each Phillips curve model considered. For example, suppose that one estimates equation (1) and finds out that $\beta_0 > 0, \gamma_1 > 0$, and $\delta_1 = 0$. Then, based on this information one can reject both the NKPC and HPC, but not the APC or SIPC.

10. Woodford (2003, p. 568) specifies an hybrid Phillips curve in the presence of habit persistence, that takes the form: $\pi_t = \beta E_t \pi_{t+1} + \kappa[(x_t - \delta x_{t-1}) - \beta\delta E_t(x_{t+1} - \delta x_t)] + u_t$. This can be written as an EPC when: $\alpha_1 = \alpha_2 = 0$, $\beta_0 = \kappa\delta$, $\beta_1 = -\kappa\delta/\beta$, $\gamma_1 = \kappa(1-\delta)(1-\beta\delta)/\beta$. Notice that a lagged change of output (marginal cost) was added as an explanatory variable. The error term of this specification is given by: $\mu_t + \kappa\delta\upsilon_t - u_{t-1}/\beta$.

11. We follow the same notation used in their paper.

If in addition one has that $\alpha_1 = \alpha_2 = 0$, then the only model consistent with data would be the SIPC.

TABLE 1
Model typology

	Model parameters				
	α_1	α_2	β_0	γ_1	δ_1
APC	-	-	+	+	0
NKPC	0	0	0	-	+
HPC	+	0	0	-	+ or 0
SIPC	0	0	+	+	0

Authors' elaboration.

4 AN ENCOMPASSING PHILLIPS CURVE FOR THE OPEN ECONOMY

The exchange rate is important in the study of inflation dynamics in open economies because it allows additional channels for the transmission of monetary policy. In an open economy, the real exchange rate will affect the relative price between domestic and foreign goods, which, in turn, will affect both domestic and foreign demand for domestic goods, and hence contribute to the aggregate-demand channel for the transmission of monetary policy. There is also a direct exchange rate channel for the transmission of monetary policy to inflation, in that the exchange rate affects domestic currency prices of imported final goods, which enter the consumer price index (CPI) and hence CPI inflation. Finally, there is an additional exchange rate channel to inflation: the exchange rate will affect the domestic currency prices of imported intermediate inputs, affecting the cost of domestically produced goods and hence domestic inflation (inflation in the prices of domestically produced goods).

We extend the EPC (equation 1) by including the level of the real exchange rate gap (q_t) in order to capture the importance of the exchange rate for inflation dynamics in an open economy:[12]

$$\Delta\pi_t = \alpha_1\Delta\pi_{t-1} + \alpha_2\Delta\pi_{t-2} + \beta_0\Delta x_t + \gamma_1 x_{t-1} + \delta_1\pi_{t-1} + \varsigma_0 q_t + \varepsilon_t. \qquad (2)$$

We call equation (2) the Encompassing Phillips curve for the Open Economy (EPCOE). Table 2 contains the implications of each Phillips curve to the signs of the coefficients of the EPCOE. Notice that the coefficient of the real exchange rate gap does not affect the model typology.

12. The level term of the real exchange rate gap can be interpreted as capturing the presence of intermediate imported goods as in McCallum and Nelson (2000). In the standard NK model, in which imports are treated as a final consumer good, inflation depends on changes of the real exchange gap. Alternative ways of introducing the real exchange rate gap in the EPCOE, like using its change (Δq_t) instead of its level (q_t), or combing both, do not change our conclusions.

TABLE 2
Model typology for the open economy

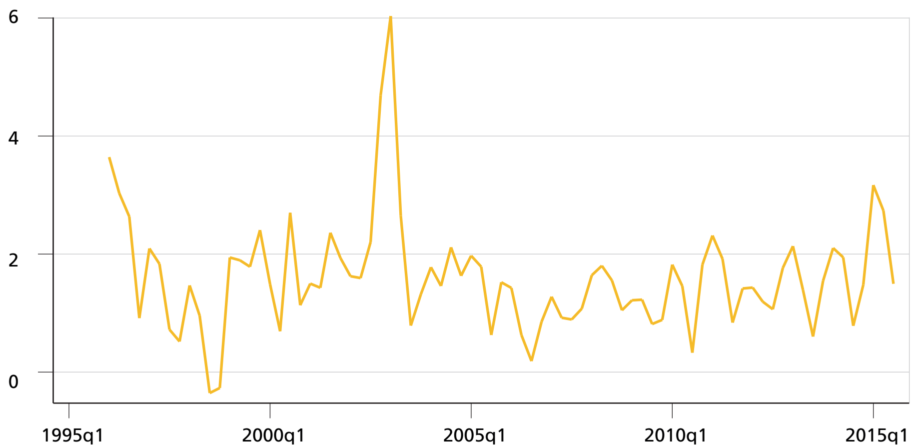
	Model parameters					
	α_1	α_2	β_0	γ_1	δ_1	ζ_0
APC	-	-	+	+	0	- , 0 or +
NKPC	0	0	0	-	+	- , 0 or +
HPC	+	0	0	-	+ or 0	- , 0 or +
SIPC	0	0	+	+	0	- , 0 or +

Authors' elaboration.

5 EMPIRICAL EVIDENCE: BRAZIL (1996Q1-2015Q3)

The sample goes from 1996Q1 to 2015Q3, the period following the real plan. Figures 1 to 3 plot the inflation rate, the output gap, and the real exchange rate gap.[13,14]

FIGURE 1
Inflation

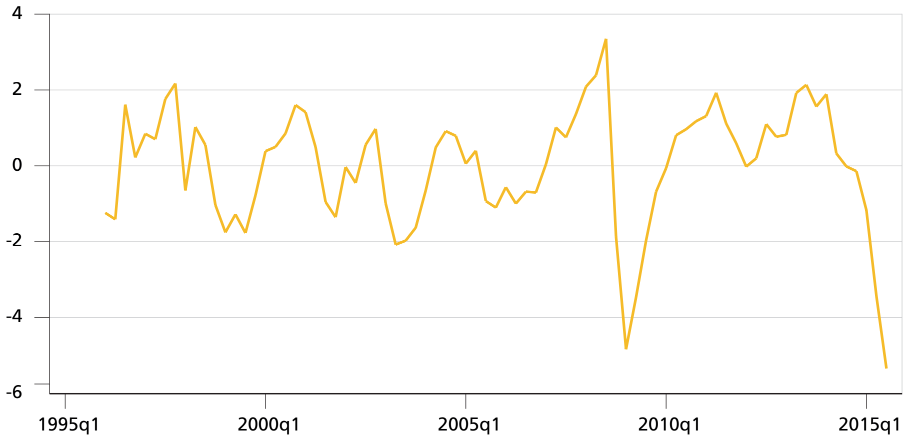


Authors' elaboration.

13. We used the Hodrick-Prescott (H-P) filter to construct our measure output gap and exchange rate gap. It is extensively employed in the literature and it represents a relatively uncontroversial way to detrend time series – see, however, Hamilton (2018). We re-did all of our GMM analyses using both log-linear detrending as well as log-quadratic detrending of the GDP and obtained results that were very similar to those that we report.

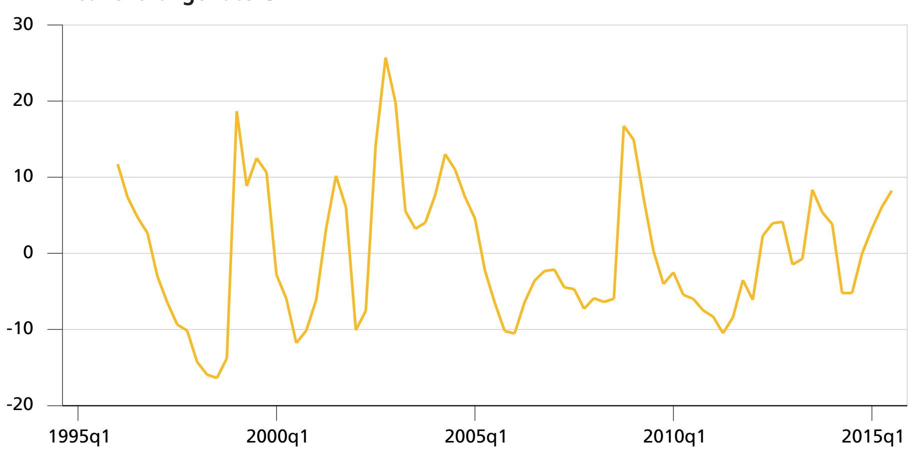
14. The data appendix (appendix A) gives details on the definitions of the variables employed in the estimations.

FIGURE 2
Output GAP



Authors' elaboration.

FIGURE 3
Real exchange rate GAP



Authors' elaboration.

Our empirical analysis starts with the EPCOE (equation 2). When the EPCOE has forward-looking elements as in the case of the NKPC and HPC, the error term becomes a function of $\varepsilon_{t\text{-}1}$, which makes the error term correlated with $\pi_{t\text{-}1}$ and $\Delta\pi_t$, by construction. A solution to the endogeneity problem lies in the use of GMM estimators. In this article we use the continuously updated (CU) GMM estimator whose estimates are independent of any normalization applied to the data. Table 3 reports CU-GMM estimates of the open-economy EPCOE on the 1996Q1-2015Q3 sample. According to table 3 the coefficients of both lagged inflation acceleration terms are negative within the 95% confidence interval, while the coefficients of the change in output gap and the output gap lag are positive. The coefficient of the inflation lag is not significantly different from zero. Using equation (2) and comparing the signs of tables 2 and 3 we observe that none of the restrictions implied by the NKPC are verified. Only two restrictions implied by the HPC are valid ($\alpha_1 > 0$ and $\delta_1 = 0$). Three of the restrictions implied by the SIPC are accepted ($\beta_0,\gamma_1 > 0$ and $\delta_1 = 0$) and two are not ($\alpha_1,\alpha_2 = 0$). All restrictions implied by the APC are accepted with the exception of the sign of the coefficient of $\Delta\pi_{t-1}$, which according to table 2 should be negative ($\alpha_1 < 0$). However, it is possible to show that the APC model is consistent with $\alpha_1 > 0$ and $\alpha_2 < 0$. Consider the APC given by

$$\pi_t = (1+\omega_1)\pi_{t-1} + \omega_2\pi_{t-2} + \omega_3\pi_{t-3} + \kappa_0 x_t + \kappa_1 x_{t-1} + \varepsilon_t,$$

in which $(1+\omega_1)+\omega_2+\omega_3=1$ and $\omega_1,\omega_3>0$.

In this model the coefficient of π_{t-1} is an overshooting mechanism that allows a cyclical fluctuation in inflation in its dynamic adjustment toward the equilibrium, in the spirit of Friedman (1971). This equation can be rewritten as

$$\Delta\pi_t = w_1\Delta\pi_{t-1} - w_3\Delta\pi_{t-2} + \kappa_0\Delta x_t + \kappa_1 x_{t-1} + \varepsilon_t,$$

since $(1 + \omega_1) + \omega_2 + \omega_3 = 1$, $\omega_1,\omega_3 > 0 \Rightarrow \omega_2 < 0$.

Therefore, all restrictions implied by the APC are accepted ($\alpha_1 > 0,\alpha_2 < 0,\beta_0,\gamma_1 > 0$ and $\delta_1 = 0$). We conclude that only the APC model appears to be consistent with inflation dynamics in Brazil from 1996Q1 to 2015Q3.

TABLE 3
Open-economy encompassing Phillips curve: GMM estimates

Dependent variable: Δπt				
Estimation method: CU-GMM				
sample: 1996Q1-2015Q3				
Variable	Coefficient	Standard error	P-value	95% confidence interval
$\Delta\pi_{t-1}$	0.278	0.101	0.006	[0.079,0.476]
$\Delta\pi_{t-2}$	-0.508	0.087	0.000	[-0.680,-0.336]
Δx_t	0.204	0.095	0.033	[0.016,0.391]
x_{t-1}	0.135	0.055	0.014	[0.027,0.243]
π_{t-1}	-0.027	0.045	0.548	[-0.116,0.061]
q_t	0.029	0.008	0.000	[0.013,0.045]
Hansen J statistic (overidentification test of all instruments): 2.533				
Chi-sq(2) P-value = 0.281				
Instrumented variables: $\pi_{t-1},\Delta\pi_{t-1},\Delta x_t$				
Included instruments: $\Delta\pi_{t-2},x_{t-1},q_t$				
Excluded instruments: $\pi_{t-2},\Delta\pi_{t-3},\Delta x_{t-1},\Delta x_{t-3},x_{t-3}$				

Authors' elaboration.

Given that the Brazilian economic policy changed in January 1999, when the exchange rate was allowed to float and additionally inflation targeting was implemented in June 1999, we tested the stability of our results by re-estimating the EPCOE for different subsamples starting after 1999Q3 and found out that our conclusions remain unchanged.

6 WEAK INSTRUMENTS

However, in order to be valid, the set of instruments chosen must satisfy two statistical conditions. First, each instrument must be uncorrelated with the error term (instrument exogeneity). Second, an instrument must be highly correlated with that portion of the endogenous regressors that cannot be explained by the other instruments (instrument relevance). When the instruments are only weakly correlated with the endogenous regressors, we have what is known as weak instruments or weak identification. Weak instruments pose considerable challenges to inference with GMM methods. If instruments are weak, then the sampling distributions of GMM statistics are in general nonnormal, and standard GMM point estimates, hypothesis tests, and confidence intervals are unreliable.

6.1 Approaches to inference with weak instruments: detecting weak instruments

One approach to dealing with weak instruments is to conduct tests of underidentification and weak identification (Baum, Schaffer and Stillman, 2007). The first diagnostic tool for assessing the strength of identification is based on a Lagrange-Multiplier (LM) test for underidentification using the Kleibergen and Paap (2006) rk statistic, see table 4. We cannot reject the hypothesis that the model is underidentified. The second set of diagnostics are based on the Stock and Yogo (2005) characterization of weak instruments using the Kleibergen-Paap Wald statistic, see table 5.[15] As the test statistic is less than the critical value tabulated by Stock and Yogo, we do not reject the hypothesis that the instruments used are weak.

TABLE 4
Underidentification test

Underidentification test
Kleibergen-Paap rk LM statistic: 4.851
Chi-sq(3) P-value = 0.183
(underidentified)

Authors' elaboration.

15. The Kleibergen-Paap Wald statistic correspond to the heteroskedasticity-robust multivariate analogue to the first-stage F statistic.

TABLE 5
Weak identification test

Weak identification test
Kleibergen-Paap rk Wald statistic: 5.018
(equation is weakly identified)

Authors' elaboration.

6.2 Approaches to inference with weak instruments: fully robust confidence regions

In order to conduct inference on the parameters of the EPCOE we use methods that are robust to weak instruments in the sense that identification of the coefficients is not assumed. This is in contrast to the conventional IV/GMM method, where the validity of tests on estimated coefficients requires the assumption that they are identified. We construct fully robust confidence regions by inverting the conditional likelihood ratio (CLR) test of Moreira (2003). Moreira's test overcomes the distortions of standard tests by adjusting the critical values for hypothesis tests from sample to sample so that, for given data, the critical values used yield a correct significance level. Thus, his critical values are "conditioned" on the data in hand, not constant. The projection-based confidence regions are obtained by grid search over the parameter space and are centered around the point estimates from the continuous-updated GMM estimator, with width set as a multiple of the Wald confidence interval.[16]

Table 6 shows the projection-based CLR confidence sets for the baseline EPCOE model where there are three endogenous regressors (π_{t-1},$\Delta\pi_{t-1}$ and Δx_t) and two exogenous regressors $\Delta\pi_{t-2}$ and x_{t-1}. Figures 4 to 6 display the scatter plots for the 2 – dimension confidence regions.[17] The results are consistent both with the view that price setting is purely backward-looking, as well as with the view that forward-looking expectations are very important in price setting. Furthermore, we do not reject the NKPC, in contrast with the findings of the previous approach, which is not robust to weak instruments.

16. To construct the fully robust confidence regions we employ the Stata routine weakiv that can estimate models with any number of endogenous regressors (Finlay, Magnusson and Schaffer, 2013).

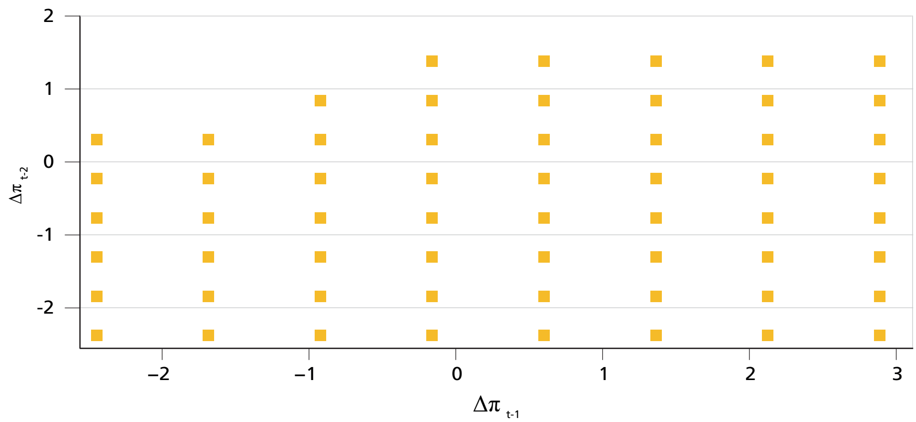
17. The confidence regions are estimated over $8^5 = 32768$ grid points.

TABLE 6
Projection-based inference

95% CLR Confidence Set	
$\Delta\pi_{t-1}$	[-2.446,2.887]
$\Delta\pi_{t-2}$	[-2.378,1.377]
Δx_t	[-2.444,2.946]
x_{t-1}	[-.976,1.385]
π_{t-1}	[-1.070,1.011]

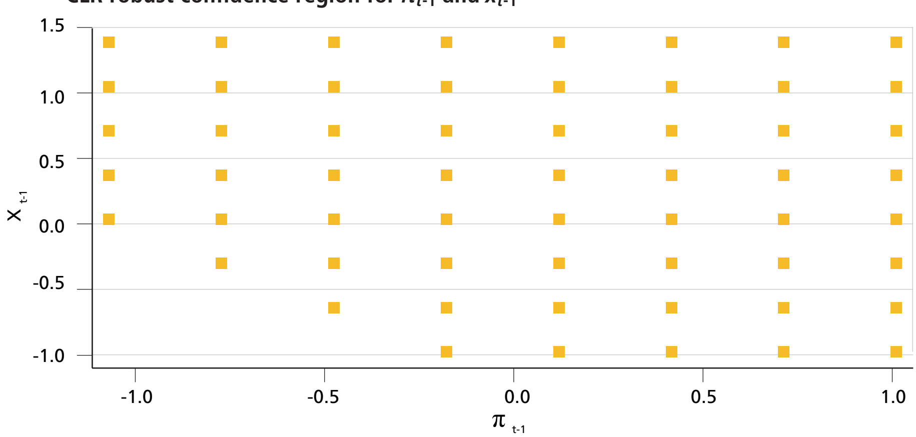
Authors' elaboration.

FIGURE 4
CLR robust confidence region for $\Delta\pi_{t-1}$ and $\Delta\pi_{t-2}$



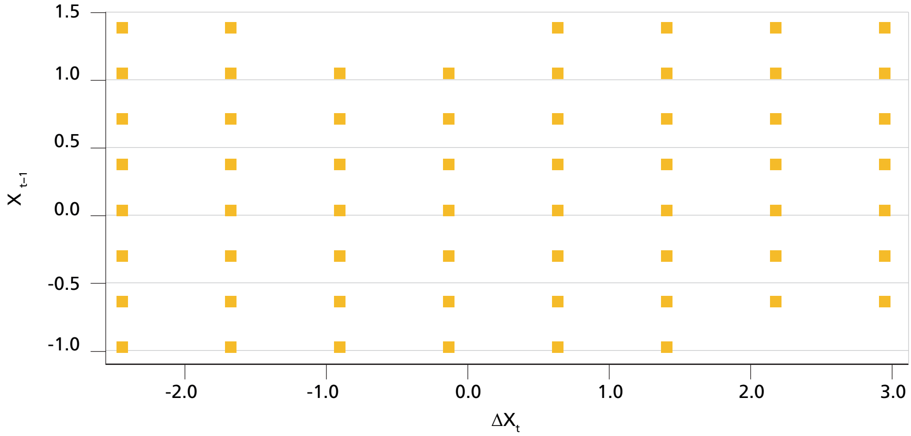
Authors' elaboration.

FIGURE 5
CLR robust confidence region for π_{t-1} and x_{t-1}



Authors' elaboration.

FIGURE 6
CLR robust confidence region for Δx_t and x_{t-1}



Authors' elaboration.

7 CONCLUDING REMARKS

In this paper, we analyzed the empirical evidence on the role of expectations in the Phillips curve using a flexible empirical approach. Our goal was to provide a clear understanding of the role of expectations that integrates across the different specifications in the literature.

Using the GMM estimator, the evidence suggests that the restrictions implied by the NKPC, HPC, and SIPC are rejected for the period after the Real Plan in Brazil. Only the restrictions implied by the APC are not rejected. However, when we construct confidence regions that are robust to weak instruments, it is not possible to reject any of the Phillips curve specifications, including the NKPC.

Using identification robust methods, we found large confidence sets for the encompassing Phillips curve parameters, which suggests that they are weakly identified. In this context, standard inference methods are unreliable and seemingly innocuous specification changes lead to big differences in point estimates. These confidence regions were constructed using projection-based methods, which are very conservative, especially when many dimensions of the structural parameter vector are projected out. To the best of our knowledge, there is currently no alternative way of making fully robust inference to weak instruments having more than one endogenous variable, as in our case. As pointed out by Mikusheva (2010), this seems extremely difficult to do. Nonetheless, we hope that this becomes a topic of research for those working at the frontier of inference with weak instruments.

We were unable to pin down the role of expectations in the inflation process sufficiently accurately for the results to be useful for policy analysis. The evidence is consistent both with the view that expectations are forward-looking, as well as with the opposite view that they are backward-looking.

The NKPC continues to be a key building block of modern macroeconomic models. But measuring the effect of expected, future inflation on current inflation can be problematic because of weak instruments. Future research on this key response would be valuable because such forward-looking effects continue to have implications for the design of good monetary policy.

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APPENDIX A

DATA APPENDIX

1) The inflation rate (π_t) is measured as the quarter-to-quarter change in the Consumer Price Index (IPCA), where the quarterly inflation rates are calculed by arithmetic averaging of the monthly series.
2) The output gap (x_t) is given by 100 times the log of the quarterly real gross domestic product (GDP) seasonally adjusted, detrended by the Hodrick-Prescott (HP) filter.
3) The real exchange rate gap (q_t) is calculated as 100 times the log of the arithmetic average of monthly indexes of real effective exchange rates, detrended by the HP filter.

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